A new method of analyzing vectors for changes in astigmatism

Noel A. Alpins, F.R.A.C.O., F.C.Ophth., F.A.C.S.

ABSTRACT

This method of astigmatism analysis recognizes the need to define an astigmatism goal, thus allowing the surgeon to obtain precise, separate measures of the magnitude and the angle of surgical error. From this, the surgeon can evaluate what surgery may be required to achieve the initial preoperative goal. An index that measures surgical success is adjusted for the level of preoperative astigmatism. The resulting data allow statistical comparison of multiple surgeries and techniques. This method also assists in resolving the case when spectacle and corneal astigmatism do not coincide.

Key Words: angle of error, astigmatism goal, astigmatism magnitude and axis, coefficient of adjustment, difference vector, index of success, magnitude of error, surgically induced astigmatism vector, targeted induced astigmatism vector, vector analysis

Current methods of analyzing astigmatism calculate the vector of change surgically induced in attaining the postoperative result from the preoperative state. This allows the surgeon to determine total induced astigmatism and the direction of the vector force acting in the eye and to calculate the mean total surgical astigmatism induced when a series of operations are compared and analyzed. However, the axes of surgically induced astigmatism (SIA) usually vary considerably within the 180-degree range of arc, making it difficult to make meaningful comparisons of astigmatic change for a series. An average directional change of vectors cannot be calculated because vectors in opposing or partly opposing directions cancel out each other in varying amounts.

In previous calculations,[1,2] patient results were tabulated individually. Hall and coauthors[1] and Merck and coauthors[2] were unable to show a trend in induced astigmatism vectors as a group because the vectors had variable orientation.

Mean of the angles does not provide a trend for axes, nor does it address the change in axes from preoperative to postoperative astigmatic status. This method does not assess the success or desirability of the result; nor does it indicate the extent to which the surgical goal was achieved. Some authors[3–6] have attempted to correct the magnitude for the degree of axis change (induced by tangential incisions) by suggesting that this component varies as the cosine of the difference between the desired and the observed (achieved) axes. This corrected value of magnitude was substituted as the amount of surgically induced cylinder 90 degrees to the axis of the incisions, the "proper" axis. Thornton and Sanders[3] programmed Naylor's[7] equations into a computer program that required slight modifications to resolve ambiguity, essentially reproducing the Naylor table.

The formula for calculating the SIA is derived from the resultant lens of two plano-cylindrical lenses with axes at different angles.[8] Researchers, including Naylor, have used the formula and confirmed the magnitude and axis of the astigmatic change using graphs. Jaffe and Clayman[9] use rectangular and polar coordinates to determine, by vector analysis, the formula for calculating SIA and its axis with the known values for preoperative and postoperative corneal astigmatism. Hall[1] derived analogous formulas based on Martin and Welford's[10] derivation of Euler's theorem of curved surfaces.

Euler's theorem, which states that "the sum of the curvatures of any two perpendicular sections of a cylindrical or toric surface has a constant value," provides the link between Jaffe and Clayman's[9] and Naeser's[11] meth-

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Reprint requests to Noel A. Alpins, F.R.A.C.O., F.C.Ophth., F.A.C.S., Cheltenham Eye Centre, 7 Chesterville Road, Cheltenham, Melbourne, Victoria 3192, Australia.

ods of vector analysis. Naeser's method calculates the polar values of astigmatism, arising when the axis of astigmatism does not lie on the 90-degree or 180-degree meridian; its use lies primarily in interpreting results of surgery that induces polar (with-the-rule and against-the-rule) changes, such as cataract and intraocular lens implantation surgery.

In a guest editorial, Thornton[12] described astigmatism as a unique refractive error that causes reduced visual acuity and produces such symptoms as glare, monocular diplopia, asthenopia, and distortion. For several years, astigmatism control and correction have been of great concern to refractive, cataract, and corneal surgeons.[13] Reduction or elimination of astigmatism, as a single or combined procedure, is possible only if one understands astigmatic change in its component parts of magnitude and axis.

Current analytical techniques do not allow meaningful separate comparison of magnitudes and axes for a series of paired groups of procedures when a change in the astigmatic status of the eye is intended. These comparisons are necessary to perfect techniques of astigmatism surgery; that is, to determine the best surgical technique and whether failure to achieve surgical goals is the result of individual patient factors or machine or technique error. Modern laser technologies allow sophisticated modification of procedures. This in turn requires analysis systems that accurately quantify and scientifically assess the results.

This article discusses a method of astigmatism analysis based on classical vector analysis techniques. It uses precise rectangular coordinates and provides the exact magnitude, angle of surgical error, and the difference vector (i.e., the magnitude and axis of the astigmatism vector required to obtain the goal from the residual astigmatism remaining after initial surgery). Concepts and terms used in the method are detailed in Appendix A.

By stating a goal for astigmatism surgery, we can calculate the vector (i.e., steepening) force required to achieve that goal. From this, we can calculate the principal components by which an operation fails to achieve its goal, as well as other components that assist in comparative analysis of astigmatism surgery.

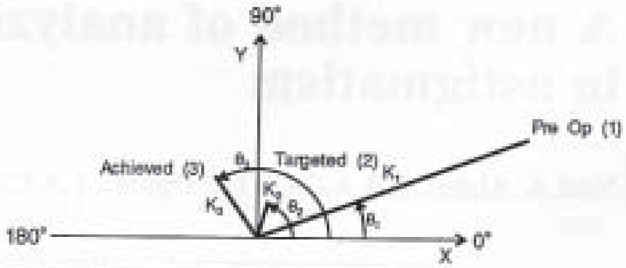
THEORY AND METHODS

Astigmatism Values

Astigmatism values (see Figure 1) used to assess results are (1) preoperative astigmatism, magnitude K_1 diopters (D) at steepest axis Θ_1; (2) target astigmatism, magnitude K_2 D at steepest axis Θ_2; (3) achieved astigmatism, magnitude K_3 D at steepest axis Θ_3, K_1, K_2, and K_3 are the dioptric differences between the steepest and flattest curvatures of the cornea, at the steepest axes Θ_1, Θ_2, and Θ_3.

Double Angles

Astigmatism is normally represented in a 0-degree to 180-degree sense. This representation complicates interpretation of results. For example, a change in astigmatism from a preoperative value of 5 degrees to a postoperative value of 175 degrees appears both graphically and numerically to be 170 degrees. It is, in fact, only a 10-degree change.



Astigmatism			
		Example	
Pre Op	K_1 Ax θ_1	4.00 Ax	20
Targeted	K_2 Ax θ_2	0.75 Ax	70
Achieved	K_3 Ax θ_3	1.25 Ax	125
Angle of correction	$\theta_3 - \theta_2$		+55

Fig. 1. (Alpins) Astigmatism diagram.

Doubling the angles (see Figures 2 to 4) ensures that results are examined in a 360-degree sense, so rectangular coordinates may be used. Doubling the angles simplifies interpretation of differences between preoperative, desired, and achieved astigmatic values and is necessary to determine the magnitude and direction of the surgical vectors.

Converting from Polar Coordinates to Rectangular Coordinates

To calculate angles and magnitudes, the polar coordinates are converted to rectangular coordinates as follows:

$$X_1 = K_1 \text{ cosine}(2\Theta_1)$$
$$Y_1 = K_1 \text{ sine}(2\Theta_1)$$
$$X_2 = K_2 \text{ cosine}(2\Theta_2)$$
$$Y_2 = K_2 \text{ sine}(2\Theta_2)$$
$$X_3 = K_3 \text{ cosine}(2\Theta_3)$$
$$Y_3 = K_3 \text{ sine}(2\Theta_3)$$

X_1, X_2, and X_3 are the X-axis coordinates on a 360-degree vector diagram and Y_1, Y_2, and Y_3 are the Y-axis coordinates.

Angle and Magnitude of Astigmatism Vectors

Figure 5 shows the target induced astigmatism (TIA), SIA, and difference vectors. The differences between the X- and Y-axis coordinates of the preoperative (1), targeted (2), and achieved (3) astigmatisms are therefore:

$$X_{12} = X_2 - X_1$$
$$Y_{12} = Y_2 - Y_1$$
$$X_{13} = X_3 - X_1$$
$$Y_{13} = Y_3 - Y_1$$

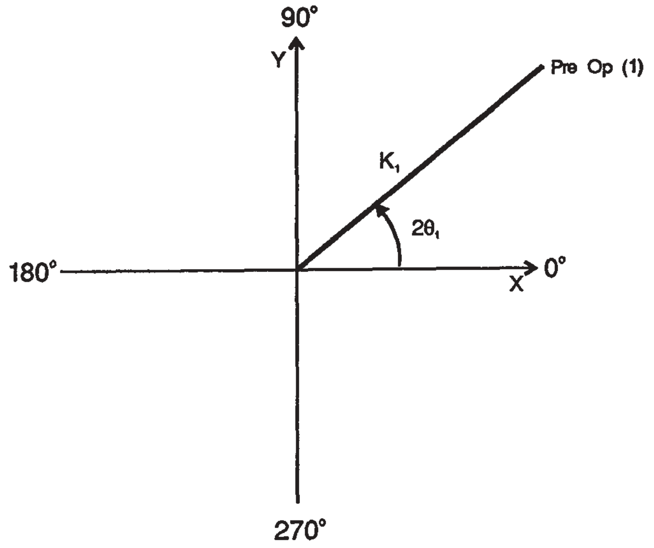


Fig. 2. (Alpins) Double-angle vector diagram.

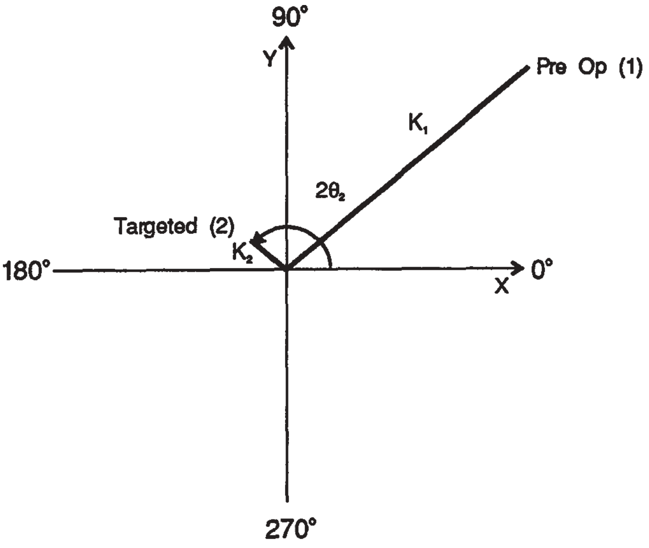


Fig. 3. (Alpins) Double-angle vector diagram.

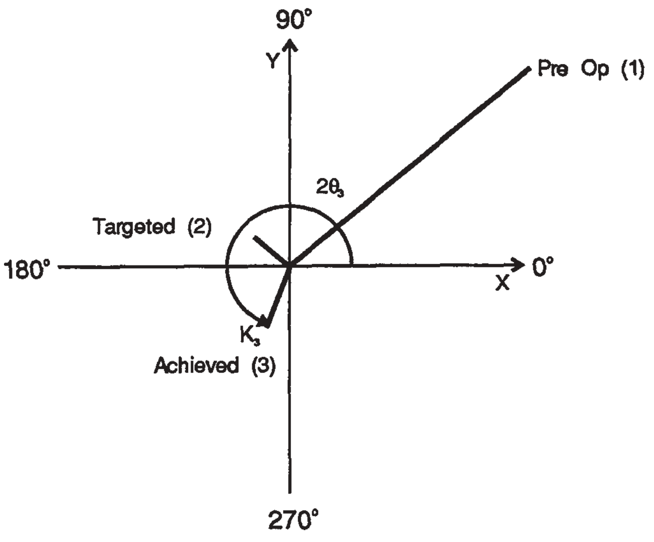


Fig. 4. (Alpins) Double-angle vector diagram.

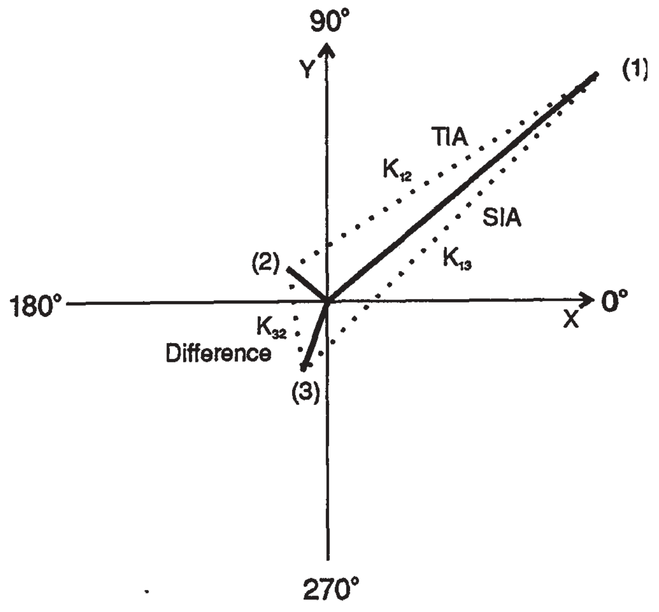


Fig. 5. (Alpins) Diagram of vector analysis method.

$$X_{32} = X_2 - X_3$$
$$Y_{32} = Y_2 - Y_3$$

These two vectors—the TIA and the difference—together with description and calculation of the various relationships of the TIA vector with the previously described[9] SIA vector, comprise the essence of this method, which has not, I believe, been previously described.

Determining Astigmatism Vector Angles

The double-angle values of the astigmatism vectors are calculated using the X and Y axis differences:

$$\Theta_{12d} = \arctan (Y_{12})/(X_{12})$$
$$\Theta_{13d} = \arctan (Y_{13})/(X_{13})$$
$$\Theta_{32d} = \arctan (Y_{32})/(X_{32})$$

The arc tangent calculation returns a value within the fourth and first quadrants; that is, it does not distinguish whether the angle is in a to–from or from–to sense. A 180-degree correction is required when the magnitude (see below) is calculated to be a negative value, as the required angle actually lies in the second and third quadrants.

The magnitude of the astigmatism vectors K_{12} (TIA), K_{13} (SIA), and K_{32} (difference vector) can now be calculated:

$$K_{12} = Y_{12}/\text{sine}(\Theta_{12d})$$

$$K_{13} = Y_{13}/\mathrm{sine}(\theta_{13d})$$
$$K_{32} = Y_{32}/\mathrm{sine}(\theta_{32d})$$

Both positive and negative values for K_{12}, K_{13}, and K_{32} are possible. Negative values indicate that the values of θ_{12d}, θ_{13d} and θ_{32d} need to be adjusted by 180 degrees. Once these corrections to the angles are made, the absolute values of the magnitudes are used.

The above method differs from that adopted by Jaffe and Clayman,[9] who used the Law of Cosines to determine the magnitude of the SIA as below (conformed for Figure 5):

$$K_{13} = (K_1^2 + K_3^2 - 2K_1K_3 \ \mathrm{cosine}2[\theta_1 - \theta_3])^{1/2}$$

The difficulty encountered using the Law of Cosines is that the sign of the value calculated is not determinable and by convention is taken as being positive (i.e., the square root of the square of −4 is evaluated as +4).

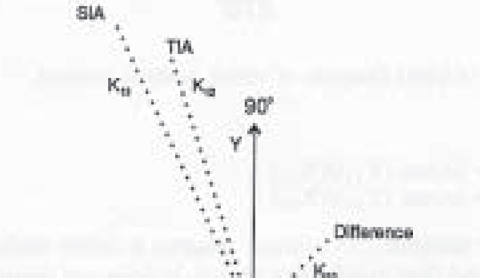
The alternative method of calculation used here to determine K_{12}, K_{13}, and K_{32} produces the same absolute value as that obtained using the Law of Cosines, but with either a positive or negative sign. A positive value indicates that the value calculated for θ_{12d}, θ_{13d}, or θ_{32d} does not require adjustment. A negative value means that the required angle is 180 degrees different from that calculated (i.e., it lies in the second and third quadrants).

If the Law of Cosines is used, additional calculations and tests are required to determine when a 180-degree correction must be made to the double-angle value of θ_{12d}, θ_{13d}, or θ_{32d}.

Halving Angles

The calculated values for the vector angles θ_{12d}, θ_{13d}, and θ_{32d} are derived from the double-angle vector diagram. The actual vector angles are of half the size (see Figure 6):

$$\theta_{12} = \theta_{12d}/2$$
$$\theta_{13} = \theta_{13d}/2$$
$$\theta_{32} = \theta_{32d}/2$$



Surgical Vectors			
		Example	
Targeted Induced Astigmatism (TIA)	K_{12}	4.20 Ax	105
Surgically Induced Astigmatism (SIA)	K_{13}	5.12 Ax	114
Difference	K_{32}	1.66 Ax	48

Fig. 6. (Alpins) Diagram of surgical vectors.

Angle and Magnitude of Error

The angle of error is positive when the SIA vector lies further counterclockwise than the TIA vector and negative if the change is further clockwise (see Figure 7). The magnitude of error is a positive value if the SIA vector is larger than the TIA vector and negative if smaller than the TIA vector.

The angle of error is most readily calculated from the double-angle values of the TIA vector and the SIA vector (Figure 5). On the 0-degree to 180-degree single-angle vector diagram (Figure 7), the angle appears as the angle between the vectors. However, if the absolute value of the θ_{error} is greater than 90 degrees, the angle is adjusted to bring it into the 0-degree to 90-degree range by adding the smaller angle to 180 degrees minus the larger angle.

The angle of error is calculated as:

$$\theta_{error} \equiv (\theta_{13d} - \theta_{12d})/2 \qquad (7)$$

The magnitude of the error is calculated as:

$$K_{error} = K_{13} - K_{12}$$

Difference Vector

The difference vector represents the amount of dioptric correction still to be induced to reach the targeted goal from the achieved result; its corresponding orientation of action is from point 3 to point 2 (Figure 5).

The angle of the difference vector is:

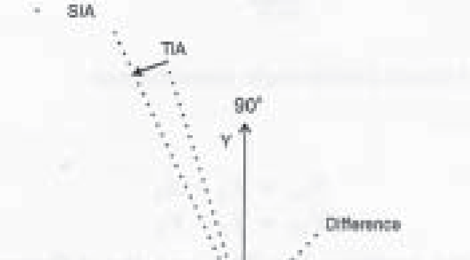
$$\theta_{diff} \equiv \theta_{32}^{d}/2$$

The magnitude of the difference vector is:

$$K_{diff} = K_{32}$$

Angle of Correction

Whereas the angle of error relates to the SIA and TIA vectors, the angle of correction deals with the achieved and targeted astigmatism. The difference between the achieved and targeted astigmatism angles is the angle of correction.



Analysis		Best
Angle of Error	+9	0
Magn. of Error	+0.92	0
Coeff. of Adjustment	0.82	1
Index of Success	0.40	0

Fig. 7. (Alpins) Analysis of surgical vectors.

The angle of correction is:

$$\theta_3 - \theta_2$$

A positive value indicates that the result is counterclockwise of the aim, and a negative value means that it is further clockwise. The value is independent of the preoperative astigmatism.

Although the angle of correction is a measure of the final astigmatic result, it is not as useful as the angle and magnitude of error values are in determining and comparing the success of astigmatic surgery.

Coefficient of Adjustment and Index of Success

An overcorrection of magnitude has occurred if the SIA vector is larger than the TIA vector and an undercorrection if it is smaller. The coefficient of adjustment adapts future astigmatism values to take account of a past trend of variance between the SIA and TIA vectors. The coefficient of adjustment is K_{12}/K_{13}.

The index of success relates to the magnitude of the difference vector to the magnitude of the TIA vector. The index of success is K_{32}/K_{12}. The index of success can only be used if an attempt has been made to induce an astigmatic change in the eye.

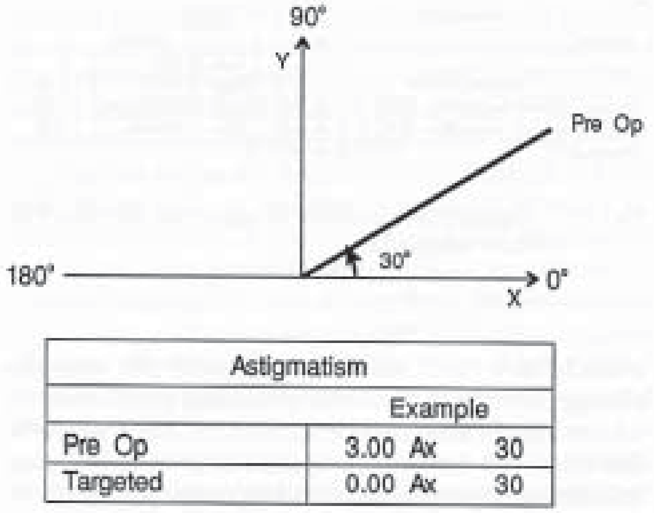
DISCUSSION

Current methods of astigmatism analysis do not allow meaningful comparisons of operations when a change in the astigmatic status of the eye is intended. To improve astigmatism surgery, we must be able to analyze less-than-perfect surgery (i.e., whether it was an error in magnitude, axis, or a combination), and we need to be able to quantify these separately. By reducing the error to its component parts, we can group separate operations together, seek trends, analyze means and standard deviations, and statistically compare techniques. Currently we can only calculate SIA that has already occurred and the changes on the polar axes. To improve astigmatism surgery we must set a specific goal for each surgical procedure, not only when a change in the astigmatic status of the eye is intended, but also when no astigmatic change is sought.

The ultimate goal of the astigmatism surgeon will likely continue to be to achieve zero astigmatism, both at the corneal plane, which is our effective site of modifying astigmatism, and at the spectacle plane, which is the secondary surface for modifying residual refractive astigmatism. Just as Naylor's[7] calculation of the power of the obliquely crossed cylinder at the spectacle plane forms the foundation for Jaffe and Clayman's[9] method for calculating the SIA at the corneal plane, this current method is as germane to the analysis of astigmatic change at the spectacle plane as it is at the corneal plane.

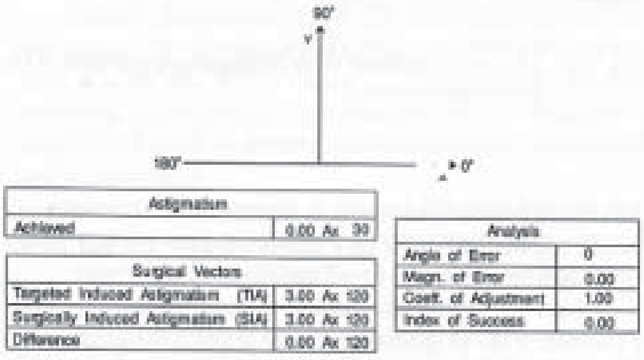
When zero astigmatism at the corneal or spectacle plane is the goal of astigmatism surgery, the 360-degree vector diagram (Figure 5) is simplified by the TIA vector overlying the preoperative astigmatism, but 180 degrees opposite in direction to the preoperative astigmatism. Any residual astigmatism is equal in magnitude to, and 180 degrees opposite in axis to, the difference vector.

Figures 8 to 15 show how, by separating the component parts of the astigmatism surgery that failed to achieve corneal sphericity, this method provides additional information to the surgeon. It is incorrect to compare magnitudes of preoperative and postoperative astigmatism without regard to their respective axes. If axes were ignored, all illustrated examples in Figures 10 to 15 (not just Figure 10) would be judged as having an identical 75% correction of magnitude. Only the example in Figure 9 would be judged as having achieved a full correction. What would appear to be a 75% correction would in fact, if oriented 90 degrees to the preoperative astigmatism axis, be a 125% correction of magnitude (Figure 11). Axis and magnitude contribute to the surgical error in varying proportions according to the orientation of the achieved astigmatism. In Figures 12 and



Astigmatism		
	Example	
Pre Op	3.00 Ax	30
Targeted	0.00 Ax	30

Fig. 8. (Alpins) Astigmatism diagram.

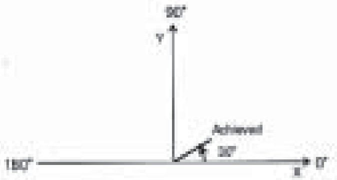


Astigmatism	
Achieved	0.00 Ax 30

Surgical Vectors	
Targeted Induced Astigmatism (TIA)	3.00 Ax 120
Surgically Induced Astigmatism (SIA)	3.00 Ax 120
Difference	0.00 Ax 120

Analysis	
Angle of Error	0
Magn. of Error	0.00
Coeff. of Adjustment	1.00
Index of Success	0.00

Fig. 9. (Alpins) Diagram of fully corrected astigmatism.



Astigmatism	
Achieved	0.75 Ax 30

Surgical Vectors	
Targeted Induced Astigmatism (TIA)	3.00 Ax 120
Surgically Induced Astigmatism (SIA)	2.25 Ax 120
Difference	0.75 Ax 120

Analysis	
Angle of Error	0
Magn. of Error	-0.75
Coeff. of Adjustment	1.33
Index of Success	0.25

Fig. 10. (Alpins) Error of magnitude of induced astigmatism.

Astigmatism	
Achieved	0.75 Ax 120

Surgical Vectors	
Targeted Induced Astigmatism (TIA)	3.00 Ax 120
Surgically Induced Astigmatism (SIA)	3.75 Ax 120
Difference	0.75 Ax 30

Analysis	
Angle of Error	0
Magn. of Error	+0.75
Coeff. of Adjustment	0.80
Index of Success	0.25

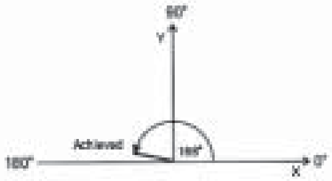
Fig. 11. (Alpins) Error of magnitude of induced astigmatism.

Astigmatism	
Achieved	0.75 Ax 72

Surgical Vectors	
Targeted Induced Astigmatism (TIA)	3.00 Ax 120
Surgically Induced Astigmatism (SIA)	3.00 Ax 113
Difference	0.75 Ax 162

Analysis	
Angle of Error	-7
Magn. of Error	0.00
Coeff. of Adjustment	1.00
Index of Success	0.25

Fig. 12. (Alpins) Error of axis of induced astigmatism.

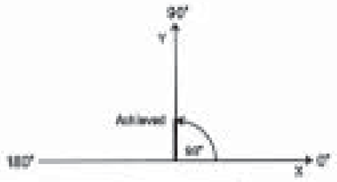


Astigmatism	
Achieved	0.75 Ax 168

Surgical Vectors	
Targeted Induced Astigmatism (TIA)	3.00 Ax 120
Surgically Induced Astigmatism (SIA)	3.00 Ax 127
Difference	0.75 Ax 78

Analysis	
Angle of Error	+7
Magn. of Error	0.00
Coeff. of Adjustment	1.00
Index of Success	0.25

Fig. 13. (Alpins) Error of axis of induced astigmatism.

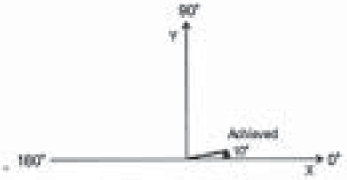


Astigmatism	
Achieved	0.75 Ax 90

Surgical Vectors	
Targeted Induced Astigmatism (TIA)	3.00 Ax 120
Surgically Induced Astigmatism (SIA)	3.44 Ax 115
Difference	0.75 Ax 0

Analysis	
Angle of Error	-5
Magn. of Error	+0.44
Coeff. of Adjustment	0.87
Index of Success	0.25

Fig. 14. (Alpins) Combined error of magnitude and axis of induced astigmatism.



Astigmatism	
Achieved	0.75 Ax 10

Surgical Vectors	
Targeted Induced Astigmatism (TIA)	3.00 Ax 120
Surgically Induced Astigmatism (SIA)	2.47 Ax 126
Difference	0.75 Ax 100

Analysis	
Angle of Error	+6
Magn. of Error	-0.53
Coeff. of Adjustment	1.21
Index of Success	0.25

Fig. 15. (Alpins) Combined error of magnitude and axis of induced astigmatism.

13, the whole of the error is attributable to treatment being applied off-axis; in Figures 10 and 11, the entire error lies in the amount of astigmatism induced.

Where the correction would appear to be lower—for example, 2.00 D postoperatively from 3.00 D preoperatively (appearing to achieve only a 33% correction)—this would, at 90 degrees to the preoperative axis, actually be a 166% correction of magnitude. Any axis changes oblique to these two meridians would result in a correction of magnitude lying between 166% and 33%, therefore with a potential range of error of 133% when calculating attempted astigmatism correction by ignoring the axis.

Comparing preoperative and postoperative astigmatism magnitudes without considering axis results in all measurements appearing as magnitude undercorrections (unless there has been a spherical result, or 100% of attempted correction with an unchanged axis). The less the apparent correction when comparing magnitudes separately, without regard to axis change, the greater the potential inaccuracy. Useful information is derived for analyzing individual and group operations, and comparing multiple surgeries and techniques only by reducing the astigmatic correction error to its component parts: magnitude and axis.

When the goal of surgery is to leave the eye's astigmatic status unaltered, the vector diagram (Figure 5) is simplified in a different way. This goal is sought in our most frequently performed operation, small incision cataract surgery, and is common in radial keratotomy and photorefractive keratectomy, where only spherical corrections are desired. With all three types of surgery, any change in the astigmatic status of the eye is unplanned and unintended, and our goal is other than zero astigmatism if the preoperative cornea is not spherical. The vector diagram (Figure 5) is simplified in that, by having the astigmatism goal overlying and coinciding with the preoperative astigmatism, the TIA vector is reduced to zero. Any SIA vector is a deviation from this coincident point and is equivalent to the magnitude of error; the difference vector is equal in magnitude and opposite in direction to the SIA vector. This simplified vector diagram is analogous to that described by Jaffe and Clayman.[9]

Targeted astigmatism correction is usually, but not always, equal to the magnitude of the preoperative astigmatism. We tend to assume that our aim is zero astigmatism, but there are instances where one would opt for a degree of residual astigmatism as the operative goal. Techniques to treat high degrees of preoperative astigmatism may not be able to produce zero astigmatism. In fact zero astigmatism is achieved in significantly less than 100% of surgery. On other occasions, such as in cataract and implant surgery, we may aim for a small amount of residual with-the-rule astigmatism to allow for the against-the-rule decay curve that is manifested in the postoperative period. In penetrating keratoplasty, where residual astigmatism is prevalent, with-the-rule astigmatism would provide better visual results.

Whether or not the goal is zero astigmatism, we can plan the axis of any residual astigmatism, whether intentionally or not. In general, any residual astigmatism lies on the same axis as the preoperative astigmatism. If residual astigmatism does deviate from the preoperative axis, it has undergone an axis shift. The surgeon could plan for the possibility that this unintended residual astigmatism will remain and will lie on the new axis and may, perhaps, correct the problem by targeting for a small amount of residual astigmatism in a preferred axis.

The astigmatic module for elliptical treatment patterns recently introduced for the excimer laser enables the surgeon to change the corneal shape in a precise and graduated manner to match the astigmatic refractive error. Accepted practice is to treat the spectacle refraction adjusted for effectivity at the corneal plane, with secondary regard to the corneal shape. There is frequently a significant variance between spectacle and corneal astigmatism, which is perplexing considering that different readings are obtained with various types of keratometers according to the optical zone measured.

The recent introduction of corneal topography technology has made this inconsistency more prevalent. To obtain meaningful data, the same type of instrument should be used for all sequential readings. Corneal topography, where available, is likely to become the preferred mode.

If the eye is treated using refraction as the treatment parameter and there is a variance between corneal and spectacle astigmatism, unavoidable non-zero corneal astigmatism will result. With astigmatic keratotomy, it is accepted practice to apply the tangential incisions at the steepest axis, with secondary regard to the refraction: the same unavoidable consequence of non-zero astigmatism is conversely destined for the refraction. It is not unusual to be satisfied with surgical outcome using keratometry readings as a criterion and yet be disappointed when the patient has such symptoms as monocular diplopia and oblique contours or still requires astigmatic correction in his or her spectacles.

The cornea is a convex surface and is steeper in its vertical meridian when with-the-rule astigmatism is present (the axis of the convex cylinder lying at 180 degrees). The clearest retinal image to this eye lies in the vertical meridian. Eggers[14] has shown that this improves visual acuity as measured by Snellen's type, as vertical strokes predominate in the English alphabet characters. Testing by a mathematical model[15] confirmed that for cases of mild myopia, viewing test objects from 0.5 to 6.0 meters, 0.50 D to 0.75 D of with-the-rule astigmatism results in the least amount of summated blur.

The nasotemporal overlap of ganglion cells that supply both optic tracts are bilaterally cortically represented. They lie on the vertical midline raphe of retinal receptors and neuronal fibers, centered on the fovea, with a width extending greater than one degree of arc.[16] This explains the much lower stereoscopic threshold for vertical objects than those orientated in any other meridian.[16] Monocular clues for determining distance are obtained by using parallax error between two objects. This is achieved most frequently with vertical contour clues (e.g., light poles).[16] In addition, the cyclodisparity range for fusion is greater for vertical than horizontal line segments.[16]

Stating and writing down goals for astigmatism surgery enables surgeons to assess success or shortcomings. By setting an astigmatic goal, we can determine how the SIA vector differs from the TIA vector. Comparative analyses of surgery using this concept of vector analysis is possible because we can determine differences and errors and thereby ascertain the correction required for future surgeries. The more accurate and predictable the surgery is, the narrower the spread of the results.

The concept of the TIA vector is the key to future astigmatism surgery using techniques such as the excimer laser. Zero astigmatism can be achieved by using a TIA vector force equal to the preoperative astigmatism and at 90 degrees to the axis of the astigmatism. The cornea is flattened in the meridian of the astigmatism, with a net steepening in the direction of the TIA vector.

It is likely that zero astigmatism will continue to be our astigmatic goal, but aiming for zero astigmatism may no longer be necessary or reasonable because of technology. Any desired postoperative astigmatism may be sought, such as 0.5 D to 0.75 D with the rule. By using the TIA vector calculated, surgery can be keyed in to the excimer laser's software program to achieve the intended corneal toroidal shape.

Non-zero astigmatism is an inevitable consequence of the conflict between a variance of spectacle and corneal astigmatism. Should the corneal shape or the refraction be the primary determinative factor in any mode of astigmatism surgery? The method discussed here helps resolve this dilemma by preoperatively assessing the least unfavorable result for the secondary surface, to which unavoidable astigmatism will be directed. This can be done by analyzing what the astigmatism consequence would be for each surface if a TIA vector were applied to achieve zero astigmatism at the other surface. The surgeon can select the TIA vector to be applied (or a suitable compromise between the two calculated) so that the refractive surface(s) to have a non-zero astigmatism is altered in the most optically and physiologically favorable orientation. The surgeon may select preoperatively the primary treatment that directs the secondary result closest to with-the-rule astigmatism, with the steepest refracting axis closest to the 90-degree meridian. Without calculating and specifying a non-zero goal, we cannot determine how successful astigmatism surgery has been.

We currently perform surgical procedures with the same astigmatic goal; that is, to eliminate the need for spectacle correction by changing the corneal shape. We are, however, performing these procedures under differing surgical paradigms. In astigmatic keratotomy (AK), we emphasize the spherical cornea; in photorefractive astigmatic keratectomy (PARK) we emphasize zero spectacle astigmatism.

In PARK surgery, by disregarding the topographical or keratometric readings, we effectively sculpt a surgically dispensed lens on the cornea without regard to the corneal astigmatism that will result. We can now calculate the targeted corneal astigmatism before surgery; as corneal surgeons, we should use the sophisticated corneal topography technology available in deciding the treatment for each individual cornea.

When planning astigmatic changes in both AK and PARK surgery, the orientation of the targeted corneal astigmatism should be the basis for what is, in fact, a surgical decision on the relative emphasis to be given to corneal and spectacle astigmatism as treatment parameters. Reducing preference for zero spectacle astigmatism will result in a reduction in targeted corneal astigmatism. The finding that effective spherical aberration is larger in the cornea with more astigmatism[17] confirms that resultant corneal astigmatism, not just spectacle astigmatism alone, should be considered in PARK surgery.

The ability to calculate the axis of the TIA vector and the angle of error accurately exposes the weakest link in our refractive surgery: the inability to identify the steepest corneal meridian precisely by real-time topography through the operating microscope during surgery. Achieving this would provide a treatment as accurate as the measurement and calculation of treatment parameters.

The method described in this article provides a mathematically precise evaluation of surgery using parameters that allow comparison between different eyes and different techniques. These parameters allow surgeons to obtain any desired level of postoperative astigmatism. By determining specific errors, we can correct each component of the error separately, resulting in better use of technology, better control, and more accurate surgery.

Vectors are like surgical navigation aids. They indicate both the direction of future surgery and the success of past surgeries. Analysis of surgical vectors effectively measures the relative success of astigmatism surgery performed by different methods and surgeons. Planning astigmatism surgery and assessing its effectiveness can only be enhanced by using surgical vectors.

REFERENCES

1. Hall GW, Campion M, Sorenson CM, Monthofer S. Reduction of corneal astigmatism at cataract surgery. J Cataract Refract Surg 1991; 17:407–414
2. Merck MP, Williams PA, Lindstrom RL. Trapezoidal keratotomy. A vector analysis. Ophthalmology 1986; 93: 719–726
3. Thornton SP, Sanders DR. Graded nonintersecting transverse incisions for correction of idiopathic astigmatism. J Cataract Refract Surg 1987; 13:27–31
4. Neumann AC, McCarty GR, Sanders DR, Raanan MG. Refractive evaluation of astigmatic keratotomy procedures. J Cataract Refract Surg 1989; 15:25–31
5. Maloney WF, Sanders DR, Pearcy DE. Astigmatic keratotomy to correct preexisting astigmatism in cataract patients. J Cataract Refract Surg 1990; 16:297–304
6. Agapitos PJ, Lindstrom RL, Williams PA, Sanders DR. Analysis of astigmatic keratotomy. J Cataract Refract Surg 1989; 15:13–18
7. Naylor EJ. Astigmatic difference in refractive errors. Br J Ophthalmol 1968; 52:422–425
8. Stokes GG. On a mode of measuring the astigmatism of a defective eye. 19th Meeting of the British Association for the Advancement of Science, 1849. Trans Sect 1850; 10
9. Jaffe NS, Clayman HM. The pathophysiology of corneal astigmatism after cataract extraction. Trans Am Acad Ophthalmol Otolaryngol 1975; 79:OP-615–OP-630

10. Martin LC, Welford WT. Technical Optics, 2nd ed, vol. 1. London, Pitman Publishing Corp, 1966; 330–331, 341–344
11. Naeser K. Conversion of keratometer readings to polar values. J Cataract Refract Surg 1990; 16:741–745
12. Thornton SP. Cataracts and the surgical control of astigmatism (guest editorial). J Cataract Refract Surg 1989; 15:11
13. Nordan LT. Quantifiable astigmatism correction: concepts and suggestions, 1986. J Cataract Refract Surg 1986; 12:507–517
14. Eggers H. Estimation of uncorrected visual acuity in malingerers. Arch Ophthalmol 1945; 33:23–27
15. Sawusch MR, Guyton DL. Optimal astigmatism to enhance depth of focus after cataract surgery. Ophthalmology 1991; 98:1025–1029
16. Moses RA, Hart WM Jr, ed. Adler's Physiology of the Eye. Clinical Application, 8th ed. St. Louis, CV Mosby, 1987; 662, 634, 663, 641
17. Seiler T, Reckmann W, Maloney RK. Effective spherical aberration of the cornea as a quantitative descriptor in corneal topography. J Cataract Refract Surg 1993; 19: 155–165

APPENDIX A

Vector Analysis: Concepts and Terms

Unlike astigmatism, vectors cannot be measured; they can only be calculated. A vector is a dynamic force that steepens the cornea in a specified direction; astigmatism is a static entity that is measurable on a toroidal surface. They are fundamentally different, and even though the two share the same units of measurement, simple arithmetic calculations between them are not valid.

Surgically induced astigmatism (SIA) vector is the amount and direction of corneal steepening that occurred in achieving the operative result from the preoperative astigmatic state. The result derived does not differ from the numeric value as calculated by Jaffe and Clayman.[9] It differs only in that this method does not use the Law of Cosines so that it is simpler to determine in which quadrant the coordinates lie.

The SIA vector is the astigmatic change induced by any surgical procedure that alters the shape of the cornea. If the SIA vector coincides with the TIA vector, the astigmatic goal of the surgery has been achieved. If corneal astigmatism were successfully eliminated, the SIA vector's magnitude would equal that of the preoperative astigmatism, and their axes would be perpendicular to each other. If no change in the corneal astigmatism were intended, the SIA vector would be equivalent to the magnitude of error.

Taking a mean of the axes of the SIA vectors is not useful. They are best represented on a scatterplot when multiple surgeries are analyzed.

Targeted induced astigmatism (TIA) vector is the amount and direction of the dioptric force required to achieve any desired astigmatic goal from any preoperative astigmatic state. When the astigmatic goal fails to coincide with the achieved result, the SIA vector and TIA vector do not coincide. They may vary in magnitude, axis, or both. These variances are the magnitude of error and the angle of error. When the SIA vector is greater than the TIA vector, an overcorrection has occurred; if less than the TIA vector, there has been an undercorrection.

The greater the ratio of the TIA vector to the difference vector, the better the result achieved and the closer the index of success approaches zero. The TIA vector is the key to planning astigmatism surgery to achieve the astigmatic goal and resolve the dilemma occurring when corneal and spectacle astigmatism do not coincide.

The difference vector represents the magnitude and axis of the difference in diopters between the desired operative result and the result achieved. The angle is half that subtended on the 360-degree vector diagram; by placing its magnitude on a 180-degree chart, it would describe the dioptric correction (the amount of steepening and its axis) required for a "top-up" operation to achieve the targeted goal for that eye. This vector is specific to the one eye in which it is calculated. Using the magnitude can measure a surgery's success and provide a valuable basis for statistical analysis between multiple operations. (This is similar to the current practice of averaging SIA[9] to determine mean total induced astigmatism for a series of eyes.)

The magnitude of the difference vector gives a measure of the total dioptric distance between the desired and the achieved results. It is independent of the TIA vector and therefore does not relate the success of surgery to the initial amount of desired correction. When the difference vector is zero, the index of success is also zero, indicating the astigmatic goal has been achieved.

Magnitude and angle of error are standardized parameters that are measurable for, and directly comparable between, a series of multiple refractive surgery procedures and can determine the trend of a particular procedure. Mean and standard deviation values can be derived, providing statistical analysis. This method separates the components of the operative error—magnitude and axis—and indicates modifications to the original surgical plan required to achieve the goal, thereby enabling improved subsequent surgery. The success of a series of operations can be assessed by determining how close the mean magnitude and angle of error are to zero.

Surgical methods currently used to alter magnitude include changing the number of T-cuts, increasing or decreasing the optical zone size, changing the length or the depth of T-cuts, and altering the dimensions of the major or minor axes of the elliptical optical zone, or modifying the dimensions and thickness of the ablatable mask in the excimer lasers using these respective techniques. For axis, the methods include changing the steepest axis by 90 degrees by correcting astigmatism in excess of the preoperative magnitude, and offsetting T-cuts from the steepest axis.

Future excimer laser techniques using the TIA vector may rotate the ellipse or the ablatable mask by a calculated amount from the steepest meridian of the cornea to achieve an astigmatic goal. This would, however, require more accurate identification of the steepest corneal axis at the time of surgery than current technology permits. Current techniques of astigmatic surgery permit the adjustment of the astigmatism magnitude in subsequent surgeries, once a trend of overcorrection or undercorrection has been detected.

Magnitude of error is the difference in length or magnitude between the SIA vector and TIA vector (Figures 5 to 7). An overcorrection has occurred if the magnitude of error is positive; an undercorrection if it is negative.

Angle of error is half the angle subtended on the vector diagram (Figure 5) by the TIA and SIA vectors at the point (1) of the preoperative astigmatism value. For example, in a series of eyes, it can determine if there is an error bias toward a consistent axis, which indicates technique or machine error.

Randomly spread error with both positive and negative signs suggests patient factors are responsible.

The sign of the angle indicates the direction in which the angle is in error. Future corrective surgical action can be adjusted accordingly.

The TIA vector and the SIA vector can be represented on a 180-degree diagram (Figures 6 and 7) by halving their respective angles. This produces the angle of error and its orientation. Here, the separation between the two vectors is the angle of error, and the correction of surgical axis direction required is from the induced toward the targeted.

Angle of correction is the angle between targeted and achieved astigmatism. The angle of correction is zero if the targeted and achieved astigmatism axes coincide; the same is true on the vector diagrams if the axes coincide on the same side of the zero coordinates. If the achieved and the targeted astigmatism differ in magnitude but coincide in axis, there is a residual difference vector, angle of error, and magnitude of error. However, distinguishing between undercorrection and overcorrection according to the relative proximity of targeted or achieved astigmatism to preoperative astigmatism does not seem to provide useful information.

The coefficient of adjustment, which measures the adjustment required to improve future surgeries, can be derived from past surgical data by dividing the TIA vector by the SIA vector. This coefficient can be averaged for a series of eyes by calculating the geometric mean by taking the logarithm of each of the sample values, determining the average of the logarithm values, and taking the antilogarithm of this average. If it varies significantly from unity, a trend is apparent. The magnitude of astigmatism can be corrected in future surgeries. Multiplying the magnitude of the preoperative astigmatism by the coefficient of adjustment produces a magnitude parameter, which indicates treatment required to obtain the optimal surgical result.

A coefficient value of one indicates that there is no magnitude of error and that there is no need to make an adjustment to future treatment. A value greater than one indicates that magnitude has been undercorrected; if the value is less than one, overcorrection has occurred.

The index of success is directly proportional to the difference vector and inversely to the TIA vector. The ratio measures the success of surgery, adjusted for the desired amount of astigmatic correction. A value of zero on the index of success indicates the surgical aim has been met; the difference vector magnitude would also be zero. If only one of the angle of error or magnitude of error is zero, the index of success figure will be a number greater than zero. The index might lie between zero and one; for example, a value of 0.2 would indicate 80% success has been achieved in attaining the surgical goal. If the index of success is one, then surgery has resulted in achieved astigmatism being equally far away from the targeted as preoperative astigmatism was. There may or may not have been an astigmatic change; either way, the patient has had surgery without improving the astigmatism. The index of success can exceed one, indicating a result worse than the preoperative state.

The index is only useful if the surgeon intends to change the eye's astigmatism. For example, in an eye that has a small amount of astigmatism associated with myopia, the surgeon may choose to induce only a spherical correction to correct the refractive error. In this case, the index of success cannot be used.