Vector Analysis Applications to Photorefractive Surgery

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One of the key factors in any successful refractive surgery practice is maximum patient satisfaction, an outcome reflected in the achievement of the full potential of uncorrected visual acuity. The majority of patients undergoing excimer laser photorefractive surgery have some amount of correctable astigmatism, whether it is associated with spherical correction or less commonly cylinder alone. The technique of vector planning is fundamental in planning for and treating and analyzing these patients. It is an invaluable tool that integrates the dual modalities of optical correction and corneal shape into the laser paradigm, allowing a customized treatment plan to be employed.

The examination of astigmatism outcomes by vector analysis has been described by a number of authors in various ways. The methodology in this chapter addresses planning of photoastigmatic refractive surgery as well as the analysis requirements of corneal and refractive astigmatism.

The refractive surgeon needs not only to address and quantify the astigmatism that is to be treated but to calculate targets to quantify errors and adjustments. Periodic and frequent analysis of results for ongoing nomogram refinement enables incremental improvement of future treatments and outcomes to be refined.

■ Why Correct Astigmatism?

1.00 D of astigmatic error will, on average, decrease uncorrected visual acuity to the level of 20/30 or 20/40, depending on its orientation

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Dr. Noel Alpins has a financial interest in the *ASSORT* vector planning and outcomes analysis software program used in this chapter.

(compared to 20/50 for 1.00 D of spherical myopia or absolute hyperopia).[1] As well as causing blurring of vision, astigmatism can cause such symptoms as distortion, glare, asthenopia, eyestrain, headaches, monocular diplopia, and squinting.

■ Vector Analysis: Concepts and Terms

Vector is a mathematical term defined by magnitude and direction. Astigmatism is also described with cylinder power (magnitude) and axis (direction) so, at face value, it fits this same description. It employs the same units of diopters and degrees, so confusion of terms can easily result.

However, the two entities have fundamentally different properties. Unlike astigmatism, which is static and measurable, vectors are dynamic and can only be calculated. As a result, simple arithmetic calculations examining astigmatism or vectors can be misleading. For example, comparing pre- and postoperative astigmatism magnitudes ignores any change in the astigmatism orientation, which, outside of quantifying the scalar change, is of limited use. It does not identify the separate "errors" of magnitude and axis. If the pre- and postoperative astigmatism is known, the vectorial change that the surgery induced on the cornea can be calculated. There are two basic methods of analyzing vectors: graphic and trigonometrical.

Graphic Analysis

Graphic analysis is represented by displaying lines of different lengths (magnitude) and direction (axis) and measuring the resultant triangle connection for magnitude and direction. For example, Figure 1 shows the addition of vector "b" to the head of vector "a," resulting in defining the

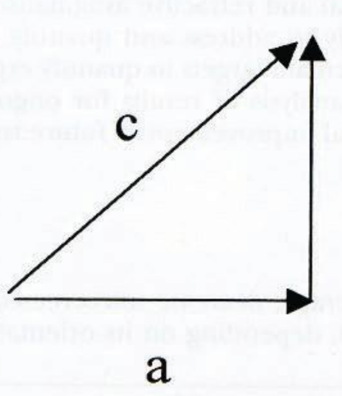


Figure 1. *Graphic analysis example.*

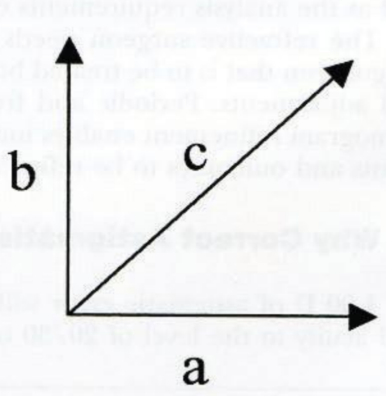


Figure 2. *Graphic analysis example.*

magnitude and direction of vector "c." Conversely, a particular vector, such as "c," can also be resolved into its components 90 degrees apart, which would achieve the same result—horizontally (180 degrees) and vertically (90 degrees—) as shown in Figure 2. A vector can be moved by transposing it to another point in space, the magnitude and direction remain the same in this case (Fig 3).

The vectorial difference in magnitude and axis between two vectors ('e' and 'f') originating from the same point in space can be determined by joining the two with a third vector ('g'), as represented by Figure 4. The above mentioned methods[2] can be used in aggregate astigmatism analysis and are described in detail later.

Trigonometrical Analysis

Trigonometric analysis is based on the fixed proportions of a right-angled triangle (Fig 5):

Sine of angle α = the opposite side to the hypotenuse divided by the hypotenuse

Cosine of angle α = the adjacent side to the hypotenuse divided by the hypotenuse

Tangent of angle α = the opposite side divided by the adjacent side to the hypotenuse

Using this trigonometric method of vector analysis, we can calculate astigmatic change in terms of power (magnitude) and axis (direction). It is important to understand that a vector, which is a steepening force, cannot have a negative magnitude. If this occurs mathematically, its direction must be reversed, by 180 degrees on a double-angle vector diagram (DAVD) or by 90 degrees on a polar diagram, to obtain a positive value. The steepening force required to correct astigmatism is equal in magni-

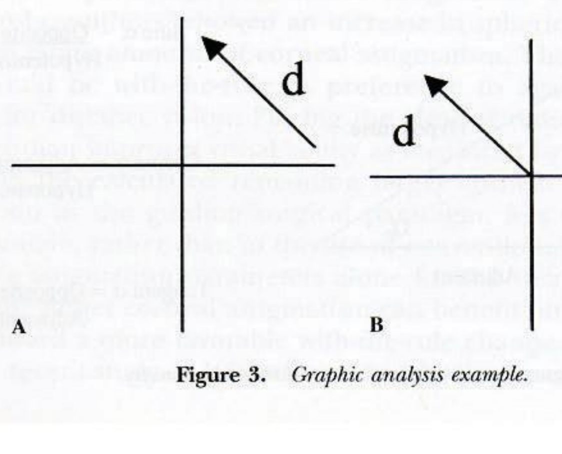


Figure 3. *Graphic analysis example.*

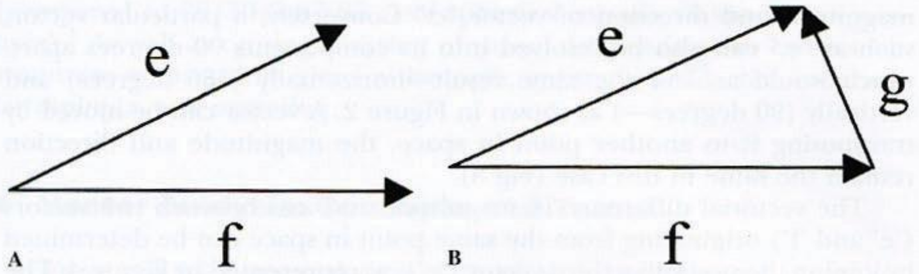


Figure 4. *Graphic analysis example.*

tude and lies perpendicular to either mode of the steepest axis (corneal measurements) and the power axis of a negative cylinder (refractive measurements).

Comparing preoperative to postoperative astigmatism without any reference to axis will provide only a vectorial analysis of the change in the cornea on certain occasions. The preoperative cornea may be spherical (no astigmatism), and surgery induces astigmatism. Further, the preoperative and postoperative axes may be absolutely aligned. Hence, in these two situations, arithmetical calculation is identical to vector calculation. In most cases, this does not apply.

To enable the use of rectangular coordinates, DAVDs are used throughout this chapter to allow calculations in a 360 degree sense and hence to simplify interpretation of changes between preoperative, targeted, and achieved astigmatic values. These calculated magnitude and axis values, which connect the arrowheads of the astigmatisms, are surgical vectors. Their axis values are then halved so that they can be represented on polar diagrams (0–180 degrees), as they would appear projected onto an eye.

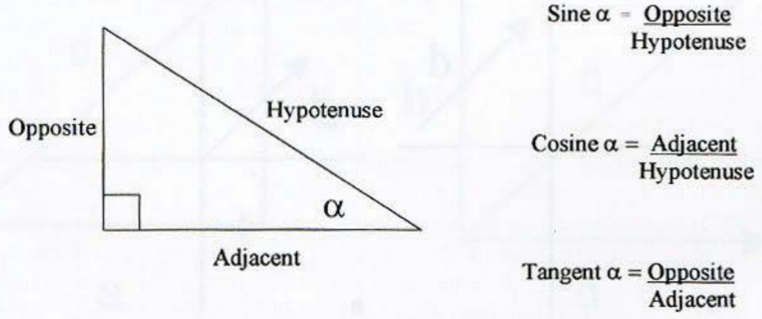


Figure 5. *Analysis of vectors using trigonometry.*

■ Vector Applications in Photoastigmatic Refractive Surgery

Let us examine how vectors can be incorporated comprehensively to address the effective treatment of astigmatism into the planning, treatment, and analyses of outcomes. The *ASSORT* outcomes analysis software program has been used to confirm detailed numerical and graphical calculations performed in this chapter.

Planning

When treating astigmatism in any form (compound myopic-hyperopic, mixed, simple myopic-hyperopic), vector planning presents surgeons with the opportunity to view the targeted outcome of a selected treatment by using both corneal and refractive values. The remaining corneal astigmatism can then be apportioned with differing emphasis to the cornea or to the refractive correction. This inclusion of corneal parameters would result in less retention of astigmatism on the cornea, with consequent reduced aberrations.

Wavefront technology has made it possible accurately to quantify higher-order aberrations in addition to the second-order spherocylindrical refractive error and third-order coma and trefoil. However, using wavefront (refractive) values alone and disregarding the corneal shape in the treatment plan can leave an excessive amount of corneal astigmatism.[3] Lower-order astigmatic aberrations and coma would not be minimized as a consequence. Variation between spectacle and corneal astigmatism is widely prevalent and, as a consequence, even at best, an inevitable amount of astigmatism remains postoperatively (known as *ocular residual astigmatism* [ORA][4,5]) in the optical system of the eye. What proportion of this unavoidable remaining astigmatism do we leave on the cornea, and what amount would remain in the manifest refraction?

The optimal result when planning such a treatment can be determined by two principles. Less corneal astigmatism is preferable to more. Seiler and coauthors[6] showed an increase in spherical aberration associated with larger amounts of corneal astigmatism. The remaining astigmatism should be with-the-rule in preference to against-the-rule (Javal's rule[4,7]) for distance vision. Placing the clearest retinal image in the vertical meridian improves visual acuity as measured by a Snellen chart.[8]

Using the calculated remaining target corneal astigmatism and its orientation as the guiding surgical paradigm, less corneal astigmatism would remain, rather than in the use of conventional treatments that use refractive astigmatism parameters alone for the treatment. This orientation of the target corneal astigmatism can benefit outcomes by providing a bias toward a more favorable with-the-rule change.

In a recent study of 100 patients undergoing laser surgery for myopia and astigmatism, 33% would have more than 1.00 D of astigmatism remaining on the cornea after surgery, and 7% would have more astigmatism postoperatively than existed preoperatively, when treated by manifest refraction values alone as treatment parameters.[4]

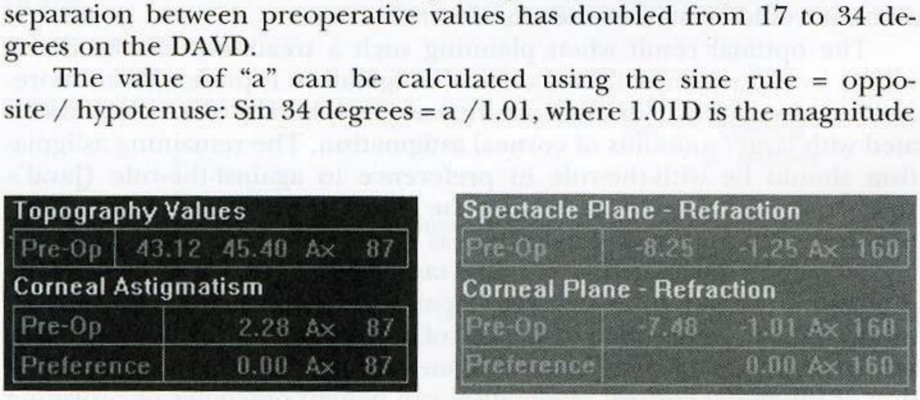
Consider a patient in whom the corneal and refractive astigmatic values are different in magnitude and direction, as shown in Figure 6. The spectacle refraction is –8.25 / –1.25 × 160, and the simulated keratometry from the topography is 43.12 / 45.40 @ 87, so that the corneal astigmatism magnitude is 2.28 D @ 87. The spectacle plane refraction has been converted to the corneal plane using a back vertex distance of 12.5 mm.

The polar diagram in Figure 7 shows the preoperative values of astigmatism for topography and refraction. Refractive astigmatism parameters are converted to the power axis (70 degrees) to enable easy comparisons between refractive and corneal astigmatism. Figure 8 shows the best achievable overall astigmatic outcome for parameters exhibited in this case: the ocular residual astigmatism (ORA) value. The ORA is the amount of astigmatism that cannot be eliminated from the eye's optical system and its correction; it is the vector difference between refractive and corneal astigmatism and is calculated (not measured) using the basic methods of analyzing vectors described earlier.[2,4] Simply put, it is the minimum amount of astigmatism that can be achieved after surgery, and it increases directly with increasing differences in preoperative corneal and refractive astigmatism magnitude and/or orientation (Figs 9, 10).

Calculating the ORA Considering the polar diagram in Figure 7, converting to a DAVD, we must double the meridia values of corneal and refractive (power axis) astigmatism (see Fig 9). Note, however, that the magnitudes remain the same.

To calculate the magnitude of the ORA vector (see Fig 10), we need to calculate values "a" and "b" (as per Figures 1, 2, and 10). Note that the separation between preoperative values has doubled from 17 to 34 degrees on the DAVD.

The value of "a" can be calculated using the sine rule = opposite / hypotenuse: Sin 34 degrees = a /1.01, where 1.01D is the magnitude



Topography Values
Pre-Op 43.12 45.40 Ax 87
Corneal Astigmatism
Pre-Op 2.28 Ax 87
Preference 0.00 Ax 87

Spectacle Plane - Refraction
Pre-Op -8.25 -1.25 Ax 160
Corneal Plane - Refraction
Pre-Op -7.48 -1.01 Ax 160
Preference 0.00 Ax 160

Figure 6. *Example of patient in whom refractive and corneal astigmatism differ in magnitude and orientation.*

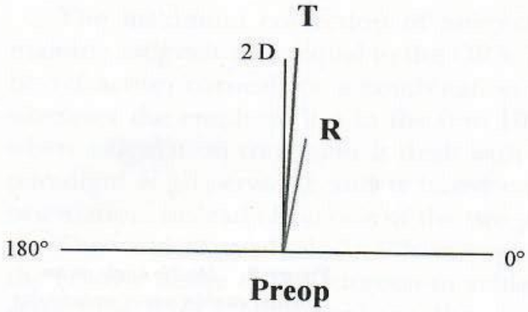


Figure 7. *Polar diagram of preoperative values of astigmatism by topography and refraction.*

of refractive astigmatism and 34 degrees (2 × 17 degrees) is the angle separating the refractive and topographic astigmatism on this DAVD.

$$a = \sin 34 \text{ degrees} \times 1.01 = 0.57 \text{ D.}$$

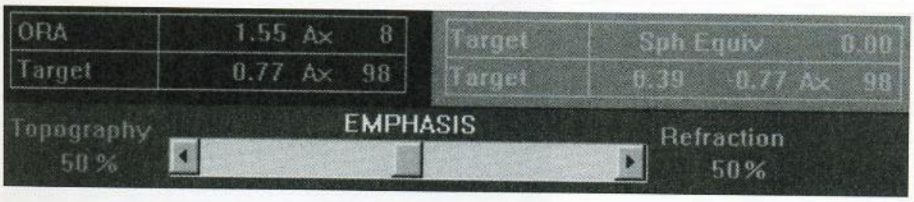
To calculate "b", we will need to determine "c" using the cosine rule and then subtract this from 2.28 D as follows:

$$\text{Cos } 34 \text{ degrees} = c/1.01 \qquad c = \cos 34 \text{ degrees} \times 1.01 = 0.84 \text{ D}$$
$$\text{Therefore, } b = 2.28\text{–}0.84 = 1.44 \text{ D}$$

Using Pythagoras's rule: $\text{ORA}^2 = a^2 + b^2$

$$\text{ORA}^2 = (0.57)^2 + (1.44)^2 = 2.39$$
$$\text{ORA} = 1.55 \text{ D}$$

Determining the Axis of the ORA To determine the angle 2θ, where θ is the axis of the ORA, we must look at its point of origin and subsequently calculate this angle from the horizontal axis (0–180 degrees); the direction of the ORA vector is *from* the head of the topography astigmatism line *to* the head of the refractive astigmatism line, due to the ORA serving as a vectorial measure of the noncorneal component of total *refractive* astigmatism or residual astigmatism as described by Duke-Elder[5];



ORA	1.55 Ax 8
Target	0.77 Ax 98

Target	Sph Equiv	0.00
Target	0.39	-0.77 Ax 98

Figure 8. *Ocular residual astigmatism and its apportionment to the corneal and refractive correction.*

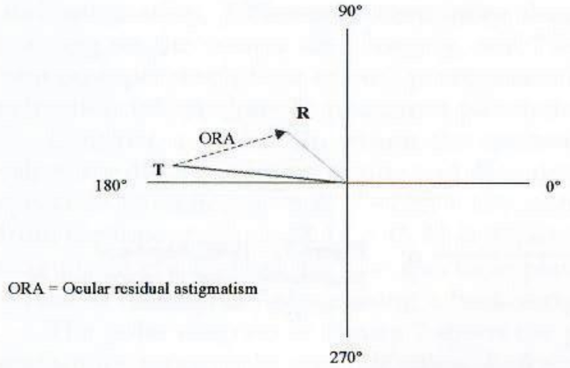


Figure 9. *Double-angle vector diagram enables use of rectangular coordinates to quantify vector differences.*

This explains that the corneal component remaining to neutralize it is at 90 degrees to it (i.e., at 98 degrees; see Fig. 8).

In Figure 10, the angle ε is equivalent to the angle 2θ and can be calculated simply as follows:

$$\sin \delta = b/\text{ORA} = 1.44/1.55 = 0.93$$
$$\delta = 68 \text{ degrees}$$

Hence, $\varepsilon = 180 - (\delta + 56 \text{ degrees} + 40 \text{ degrees}) = 16 \text{ degrees} = 2\theta$.

To convert back to polar coordinates, we must halve the angle:

$$\text{Axis of ORA, } \theta = 8 \text{ degrees}$$

$$\text{ORA (vector)} = 1.55 \text{ D Ax } 8$$

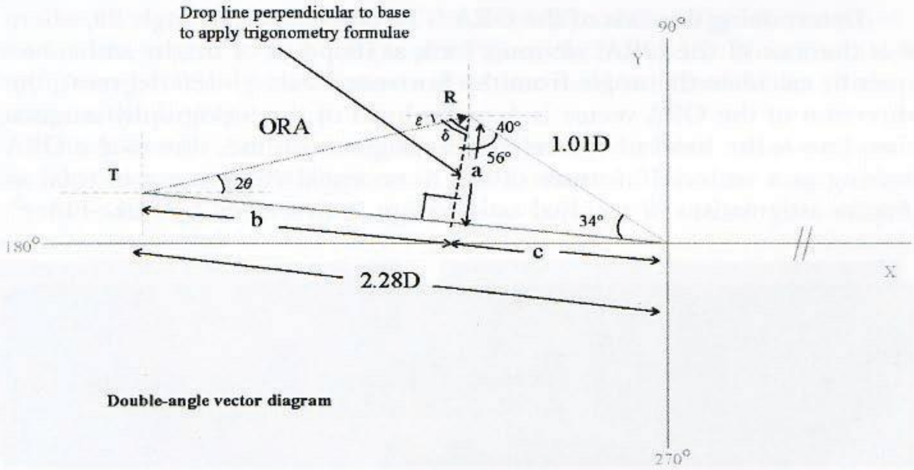


Figure 10. *Calculation of ocular residual astigmatism magnitude and axis using trigonometrical formulae.*

The maximum correction of astigmatism is achieved when the remaining astigmatism is equal to the ORA. This remaining astigmatism will be refractive, corneal, or a combination of the two and at a minimum wherever the emphasis lies in the 0 to 100% range (Fig 11). As a result, when astigmatism treatment is dealt with in this manner, one treatment paradigm is all-pervasive and is based on the astigmatism *result* and its orientation, instead of on one of the two preoperative astigmatism values.

The surgical emphasis (in this example, 50%; see Fig 8) will determine the relative desire of the surgeon to achieve either a spherical cornea or refraction. With a 50% emphasis, the topographical target is 0.77 D at a meridian of 98 degrees, and the target refraction values for a 0.00 spherical equivalent are a 0.39 D sphere, a –0.77 D cylinder, and an axis of 98 degrees (displayed at power axis of 8 degrees in Figure 11).

To treat "optimally," as the topography target meridian approaches the favorable with-the-rule orientation, more emphasis is given to sphericizing the refraction and vice versa. It is easily shown that optimal treatment closer to the principal corneal meridian would produce less corneal astigmatism[9] than treatment based solely on refractive astigmatism. The reason for this is less "off-axis" effect, which results in retention of less corneal astigmatism and, hence, less astigmatic second-order aberrations as a consequence.

The ASSORT program calculates this increase or decrease in emphasis (left to right shift; see Fig 8) according to a linear relationship with appropriate balancing of target astigmatism priorities. Figure 13 indicates how the target results are derived, and Figure 11 displays their orientation on a polar diagram.

The target refraction and topography are oriented at 180 degrees to each other on a DAVD (see Fig 13), so they form a straight line the length of which is equal to the ORA. As the shortest distance between two points is a straight line, the sum of their magnitudes is at a minimum for the eye. If the combined magnitude of the remaining astigmatism is greater than the initial ORA, the surgery has failed to achieve the maximum astigmatic

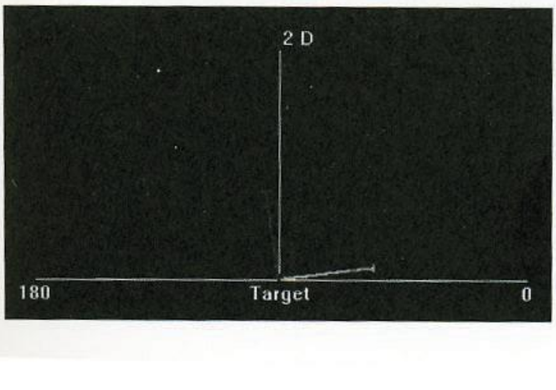


Figure 11. *Topographical refractive target values displayed on polar diagram.*

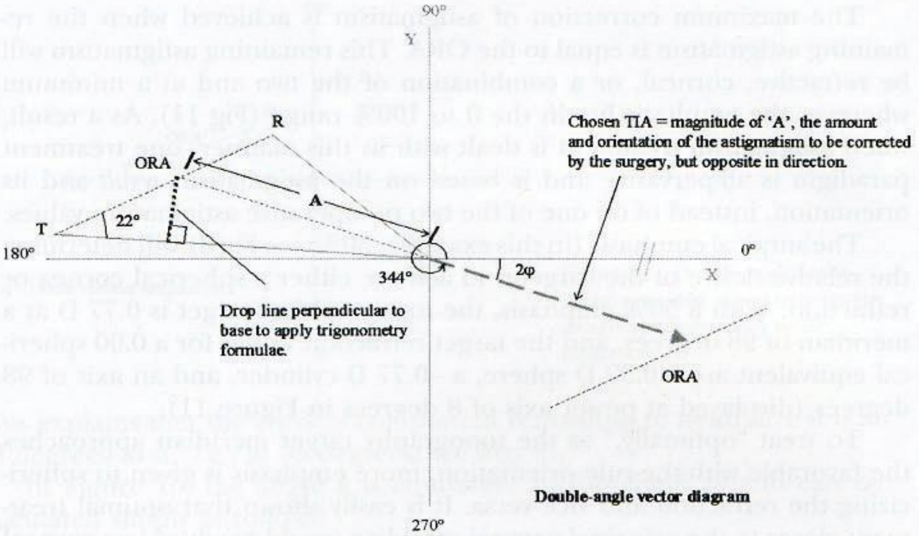


Figure 12. *Determining the magnitude and orientation of the treatment vector (TIA).*

reduction effect. This occurs if the chosen TIA does not terminate on the ORA line so that the astigmatism is being "under," or "over" treated.

Treatment

Calculating the TIA and its Orientation To achieve the abovementioned targets, the treatment necessary to reduce the corneal and refractive astigmatism maximally is calculated using the trigonometrical formulae described above and is known as the *target induced astigmatism vector* (TIA). Figure 12 shows the treatment vector with an equivalent magnitude to 'A' but 180 degrees away from the orientation of "A," the flattening effect intended on the DAVD. The actual steepening effect on the cornea would be 90 degrees away, indicating the amount of change and orientation required maximally to reduce corneal and refractive astigmatism, since it is not possible to eliminate both. The relative proximity of the TIA to either 100% correction of topographical astigmatism, or 100% correction of refractive astigmatism is determined as the emphasis of the treatment required for the respective complete correction (see Fig 13). The TIA that achieves the minimum target astigmatism for the prevailing topographical and refractive parameters must terminate on the ORA line. If not, more astigmatism (corneal plus refractive) remains than is necessary or achievable. The surgical vectors to treat optimally, 100% refraction or 100% topography are shown in Figure 14. Note that the "optimal" TIA vector lies between the refractive and topographical TIA as displayed in Figure 14 and that its length ends on a parabolic loop arching between them.

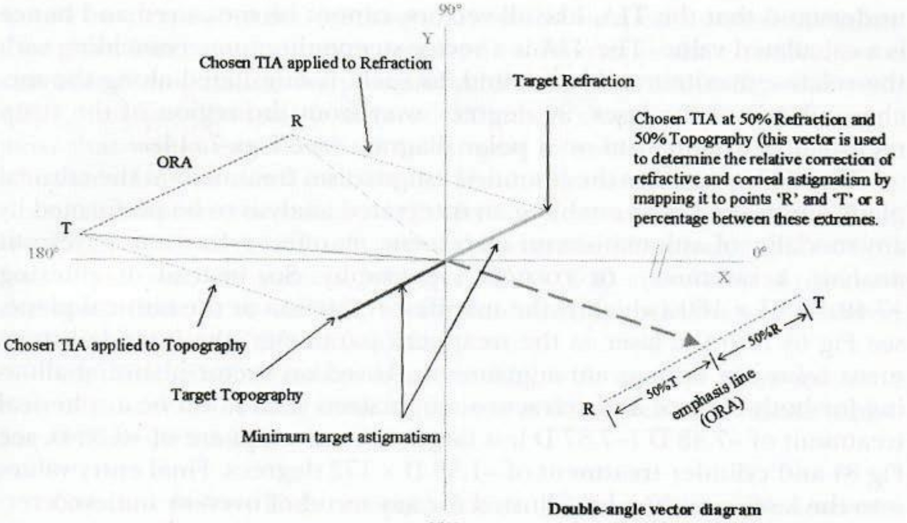


Figure 13. *The target-induced astigmatism (TIA) chosen lies between the boundaries of the topographical correction TIA and refractive correction TIA. The relative proximity of the intersection to either the topographical or refractive endpoints is determined by the emphasis of treatment, which in the example used is 50%. Any TIA that achieves the minimum target astigmatism for the prevailing topographical and refractive parameters will terminate on the emphasis line defined by the ocular residual astigmatism between complete correction of T or R.*

The calculated TIA magnitude is 1.59 D axis 172 degrees (the TIA axis of 344 degrees [angle 2φ = 360 degrees – 16 degrees] on a DAVD), halved to convert to a polar vector diagram to determine the actual orientation of the vector as it would relate to an eye (see Fig 14). It is fundamental to

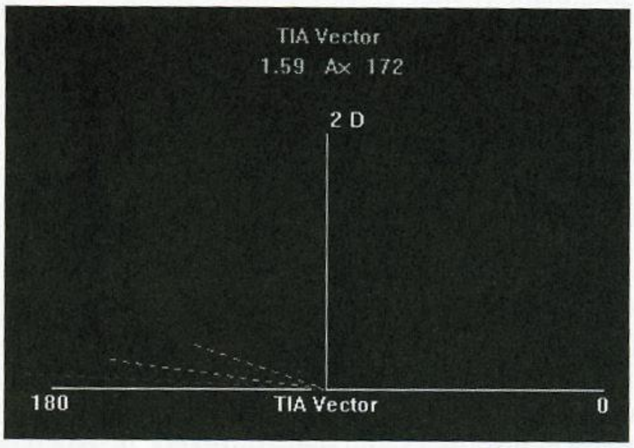


Figure 14. *The "optimal" target induced astigmatism vector (TIA) lies between the refractive and topographical TIA.*

understand that the TIA, like all vectors, cannot be measured and hence is a calculated value. The TIA is a vector-steepening force coinciding with the relative maximum ablation and, as such, is orientated along the mechanical axis of the laser, 90 degrees away from the region of the steep meridian of astigmatism on a polar diagram (see Figs 7, 14).

The TIA quantifies the intended astigmatism treatment at the corneal plane and is the key to enabling an integrated analysis to be performed by any modality of astigmatism measurement: manifest refraction, wavefront analysis, keratometry, or corneal topography. So, instead of entering –7.48/–1.01 × 160 (which is the manifest refraction at the corneal plane, see Fig 6) into the laser as the treatment parameter, the intended treatment *before* any nomogram adjustments, based on vector planning allowing for both corneal and refractive astigmatism values, will be a spherical treatment of –7.48 D (–7.87 D less target refraction sphere of +0.39 D, see Fig 8) and cylinder treatment of –1.59 D × 172 degrees. Final entry values into the laser can then be adjusted for any trend of over- or undercorrection in spherical and cylindrical treatment as well as hyperopic or myopic shifts found by analyses of previous surgeries.

Analyses of Outcomes

Postoperative Analyses: How Successful Was the Surgery? Comparing pre- and postoperative astigmatism magnitude values ignores any change in the astigmatic axis and consequently is misleading, because it inevitably renders all imperfect corrections to be "undercorrections."[10] It does not identify the separate "errors" of magnitude and axis. The Alpins method[2–4,9–11] provides the ability to compare separately the magnitudes and axes of astigmatic change (vectors) where changes in astigmatism are planned. To explain readily the corneal changes occurring after astigmatic surgery, this technique of vector analysis quantifies whether too much or too little treatment was applied and whether the treatment was on-axis or off-axis. In addition, vector analysis tools allow us to determine what proportion of the treatment was useful in reducing astigmatism and how much of the surgically induced astigmatism (SIA) resulted in undesired rotation of existing astigmatism (known as *torque*).[9] The essential process of nomogram calculation for future treatments can then be put in place.

The surgically induced astigmatism *vector* (SIA) is the amount and axis of astigmatic change that the surgery actually induced. This can also occur even when only spherical treatments are attempted (i.e., the TIA = 0) and is often responsible for an increase in induced astigmatism with an increase in spherical myopia (Tabin and coworkers[12]). When treating corneal astigmatism, if it were successfully eliminated, the SIA vector's magnitude would equal that of the preoperative astigmatism, and its axis would be perpendicular (orthogonal) to the steepest corneal meridian. The third fundamental vector required to determine the effectiveness of correcting astigmatism by photorefractive surgery is the *difference vector* (DV), best described as the induced astigmatic change (by magnitude and axis) that would enable the initial surgery to achieve its intended target whether it be 0 or not. The DV is an absolute measure of success—(i.e., it does not indicate the initial "difficulty" or amount of treatment). So, if treating 5.00 D or 1.00 D of astigmatism resulted in identical DVs, both have equal absolute success; however, the 5.00 D correction is obviously the more difficult to correct, so a greater relative success was achieved. With all surgeries, a DV of 0 is ideal.

The various relationships between TIA, SIA, and DV allow examination of outcomes of astigmatic treatment and are well described employing certain useful indices.

Correction Index The correction index (CI) is calculated by determining the ratio of the SIA to the TIA and is preferably 1.0. It is greater than 1.0 for an overcorrection and less than 1.0 for an undercorrection. This is not a measure of astigmatism remaining after surgery; the ratio value is a relative (not absolute) measure of the astigmatism induced to the intended astigmatism change.

Index of Success The success index (IOS) is calculated by determining the ratio of the DV to the TIA. The ratio measures the success of the surgery in relative terms, adjusted for the desired amount of astigmatic correction. A value 0 on the IOS indicates that the surgical aim has been met, with a 0 value for both angle and magnitude of error (ME); the DV magnitude would also be 0, and the CI would be 1.0. If only the angle of error or ME is 0, the IOS figure will be a number greater than 0. The index might lie between 0 and 1; for example, a value of 0.2 would indicate that 80% success has been achieved in attaining the surgical goal. If the IOS is 1, surgery has resulted in are uncorrected astigmatism equally "distant" from the target as the change attempted by the initial astigmatism treatment. Whatever change in astigmatism has occurred, there has been no overall improvement. The IOS can exceed 1, indicating a result worse than that in the preoperative state.

Note that a CI of 1.0 does not mean complete success, since misalignment (angle of error) between the SIA and TIA will leave a remaining DV; consequently, the IOS will also be greater than 0. These three indices all take into consideration the initial (preoperative) complexity of the treatment by incorporating the TIA into their relative values.

Individual Patient Vector Analysis A valid analysis between corneal and refractive values is achieved by converting all refractive astigmatism values to the corneal plane and performing all calculations on these corneal plane values. The right eye of a patient who had undergone LASIK surgery using the B&L Technolas laser was individually analyzed to determine changes in astigmatism. The preoperative refraction as measured by wavefront aberrometry (Zywave) was –2.24 /–2.97 × 53 at the corneal plane (Fig 15A) and –2.32/–3.33 × 53 at the spectacle plane using a back vertex distance of 15 mm (Fig 16A).

The postoperative Zywave refraction, measured at 3 months, was +1.37/–0.22 × 170 at the corneal plane (see Fig 15B) and +1.34/–0.21 × 170 calculated out to the spectacle plane (see Fig 16A; back vertex distance = 15 mm). The refractive astigmatism value is quantified by the second-order wavefront analysis spherocylindrical value. Simulated keratometry values from the Orbscan were 42.40/44.00 @ 134 preoperatively (Fig 17A) and 38.2/39.60 @ 93 at 3 months postoperatively (see Fig 17B). The treatments in this example were solely determined by refractive astigmatism values to eliminate myopic sphere and cylinder and to achieve a plano refraction. The treatment applied to the cornea is the Zywave refraction (in negative cylinder form) at the corneal plane. The preoperative corneal plane refraction is –2.24/–2.97 A×53; hence, the astigmatic treatment (TIA) is 2.97 A×53, intending to induce 2.97 D of relative steepening at a corneal meridian of 53 degrees to achieve a refractive astigmatic target of 0.00 D.

The calculated corneal target is 1.53 D @ 62 when this TIA of 2.97 D A×53 is applied to a corneal astigmatic value of 1.60 D @ 134. Note that very little reduction of astigmatism is targeted, but a large clockwise (CW) swing in its orientation is being induced. The "achieved" values as shown in Figure 16B are the measured postoperative refractive (wavefront) and corneal (Orbscan) astigmatism, respectively. The SIA is the calculated vectorial change between postoperative and preoperative astigmatism and is 3.10 D (refractive) and 1.97 D (corneal). Figure 16A and B each contain three graphical displays.

The polar diagram of astigmatic values displays the power meridian of the negative cylinder (i.e., the positive cylinder axis) for ease of comparison with the corneal astigmatism values. The DAVDs show these astigmatism magnitude values as a continuous line at twice their axis value and the respective surgical vectors as a dashed line connecting these astigmatism displays. The surgical vector polar diagrams show these same surgical vectors at their actual orientation, as they would appear on the eye, which is one-half of their axis value on the DAVD display. The values of these calculated surgical vectors are tabulated in the box adjacent to the display.

The "analysis" box reveals the effectiveness of the astigmatism procedure by its individual components: AE, ME, CI, and IOS. The refractive analysis shows a minimal overcorrection (CI = 1.04) of astigmatism, while the corneal analysis shows an undercorrection (0.67) as displayed by the CI value. This is reflected in the ME, which shows an overcorrection by refractive values (0.13 D) and an undercorrection in astigmatism by cor-

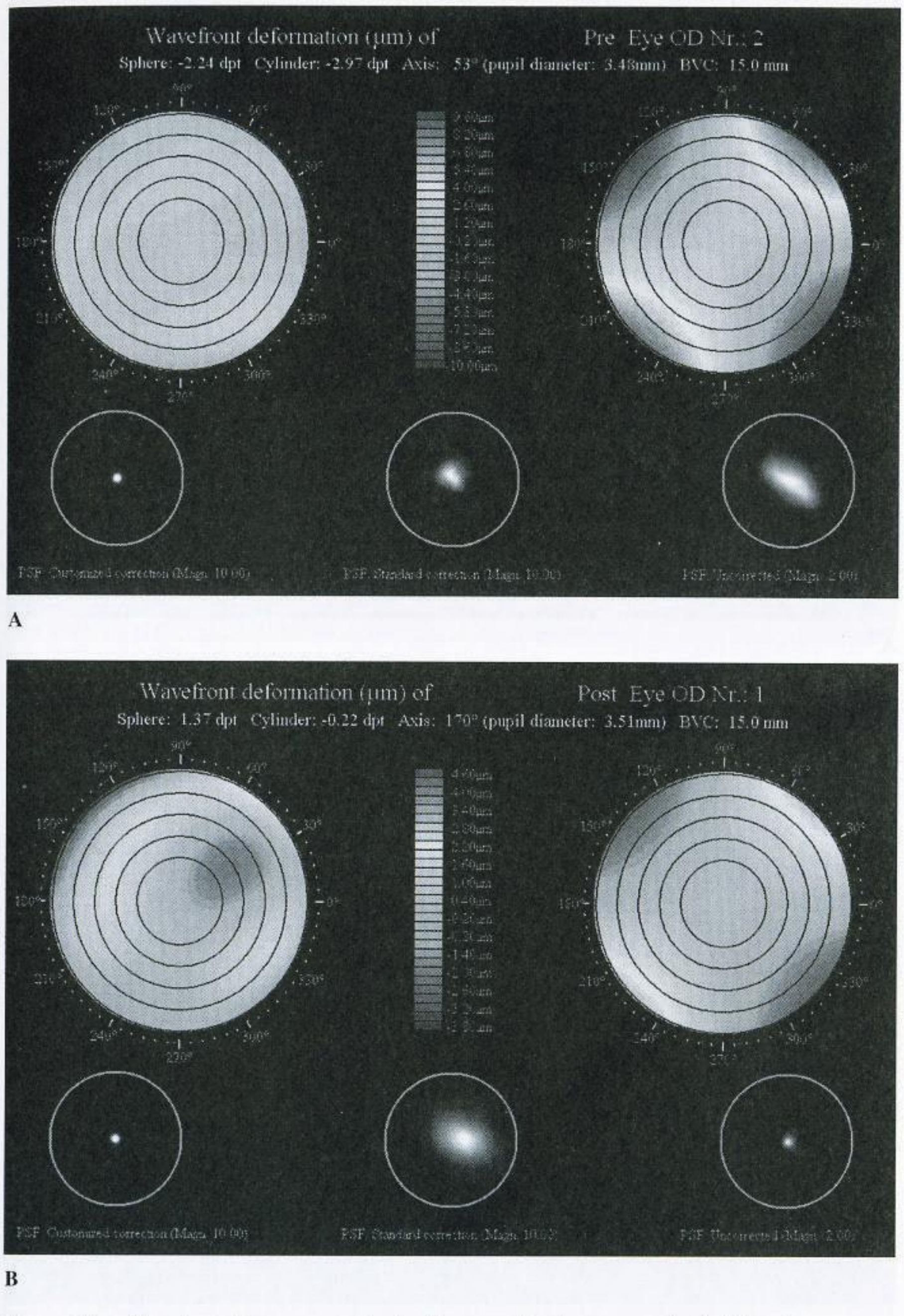
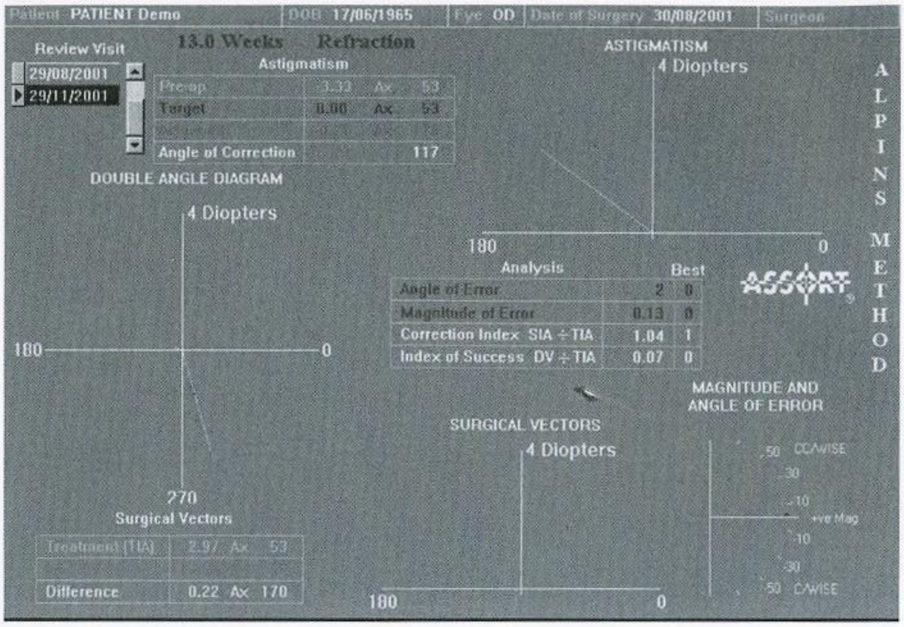


Figure 15. *Wavefront aberrometry producing Zywave refraction preoperatively (A) and postoperatively (B).*



Patient PATIENT Demo | DOB 17/06/1965 | Eye OD | Date of Surgery 30/08/2001 | Surgeon

13.0 Weeks Refraction

Review Visit
29/08/2001
29/11/2001

Astigmatism

Pre-op	3.33	Ax	53
Target	0.00	Ax	53
[illegible]	[illegible]	[illegible]	[illegible]
Angle of Correction			117

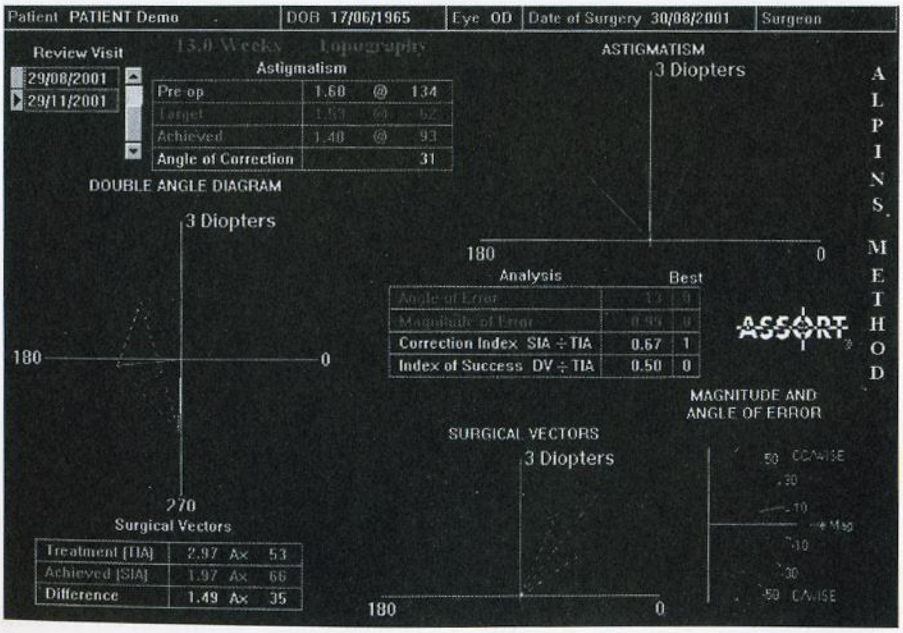
Analysis

		Best
Angle of Error	2	0
Magnitude of Error	0.13	0
Correction Index SIA ÷ TIA	1.04	1
Index of Success DV ÷ TIA	0.07	0

Surgical Vectors

Treatment (TIA)	2.97	Ax	53
Difference	0.22	Ax	170

A



Patient PATIENT Demo . | DOB 17/06/1965 | Eye OD | Date of Surgery 30/08/2001 | Surgeon

13.0 Weeks Topography

Review Visit
29/08/2001
29/11/2001

Astigmatism

Pre op	1.68	@	134
Target	[illegible]	@	62
Achieved	1.40	@	93
Angle of Correction			31

Analysis

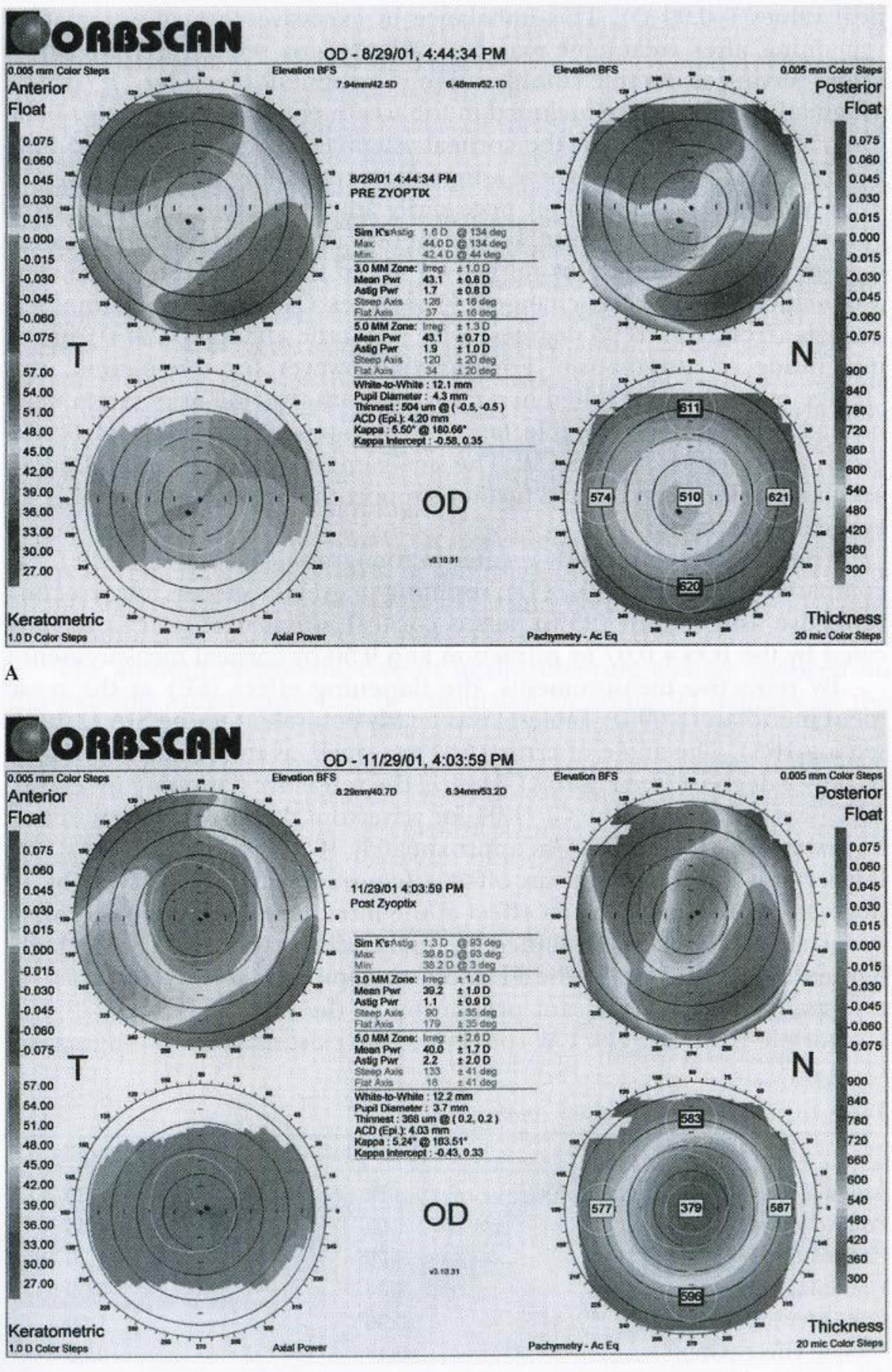
		Best
Angle of Error	[illegible]	[illegible]
Magnitude of Error	[illegible]	0
Correction Index SIA ÷ TIA	0.67	1
Index of Success DV ÷ TIA	0.50	0

Surgical Vectors

Treatment (TIA)	2.97	Ax	53
Achieved (SIA)	1.97	Ax	66
Difference	1.49	Ax	35

B

Figure 16. *(A) Individual analysis of refractive astigmatism as measured by wavefront aberrometry. (B) Individual analysis of topographical astigmatism as measured by the Orbscan device.*



OD - 8/29/01, 4:44:34 PM

7.94mm/42.5D 6.48mm/52.1D

8/29/01 4:44:34 PM
PRE ZYOPTIX

Sim K's Astig 1.6 D @ 134 deg
Max 44.0 D @ 134 deg
Min 42.4 D @ 44 deg
3.0 MM Zone: Irreg ± 1.0 D
Mean Pwr 43.1 ± 0.8 D
Astig Pwr 1.7 ± 0.8 D
Steep Axis 128 ± 16 deg
Flat Axis 37 ± 16 deg
5.0 MM Zone: Irreg ± 1.3 D
Mean Pwr 43.1 ± 0.7 D
Astig Pwr 1.9 ± 1.0 D
Steep Axis 120 ± 20 deg
Flat Axis 34 ± 20 deg
White-to-White : 12.1 mm
Pupil Diameter : 4.3 mm
Thinnest : 504 um @ (-0.5, -0.5)
ACD (Epi.): 4.20 mm
Kappa : 5.50° @ 180.66°
Kappa Intercept : -0.58, 0.35

OD

v3.10.31

OD - 11/29/01, 4:03:59 PM

8.29mm/40.7D 6.34mm/53.2D

11/29/01 4:03:59 PM
Post Zyoptix

Sim K's Astig 1.3 D @ 93 deg
Max 39.8 D @ 93 deg
Min 38.2 D @ 3 deg
3.0 MM Zone: Irreg ± 1.4 D
Mean Pwr 39.2 ± 1.0 D
Astig Pwr 1.1 ± 0.9 D
Steep Axis 90 ± 35 deg
Flat Axis 179 ± 35 deg
5.0 MM Zone: Irreg ± 2.6 D
Mean Pwr 40.0 ± 1.7 D
Astig Pwr 2.2 ± 2.0 D
Steep Axis 133 ± 41 deg
Flat Axis 18 ± 41 deg
White-to-White : 12.2 mm
Pupil Diameter : 3.7 mm
Thinnest : 368 um @ (0.2, 0.2)
ACD (Epi.): 4.03 mm
Kappa : 5.24° @ 183.51°
Kappa Intercept : -0.43, 0.33

OD

v3.10.31

Figure 17. *Orbscan topography preoperatively (A) and postoperatively (B).*

neal values (–0.99 D). This imbalance in excessive corneal astigmatism remaining after treatment may be attributed to refractive (wavefront) values serving as the sole consideration in treatment planning.[2] If corneal astigmatism values were included in the treatment plan using vector planning, as described earlier, the corneal astigmatism reduction would have been greater, with less corneal astigmatism remaining after surgery.

This could occur without necessarily adversely affecting the postoperative refractive astigmatism, so that an overall greater reduction in astigmatism (topographical plus refractive) would have been achievable. The astigmatic meridian has changed 41 degrees CW in the topographical analysis (from 134 to 93 degrees) with very little change (0.20 D) in the magnitude of astigmatism. Fortunately however for the patient, this change in meridian resulted in a favorable with-the-rule orientation @ 93 instead of its not-so-favorable targeted or intended meridian of 62 degrees, a further 31 degrees CW. The small amount of associated refractive astigmatism measured would further support the beneficial value of WTR corneal astigmatism.

The DV was significantly greater in topographical analysis (1.49 D) as compared to refractive (0.22 D), resulting in greater success in correcting refractive astigmatism (93%) versus corneal astigmatism (50%) as indicated by the IOS = 0.07 by refraction and 0.50 by corneal measurements.

By refractive measurements, the flattening effect (FE) at the treatment meridian (3.09 D; Table 1) was nearly equivalent to the SIA (3.10 D; see Fig 16A). The angle of error (AE) was small, as treatment was off-axis by only 2 degrees (see Fig 16A). Hence, the refractive flattening index (FI; 1.04) was the same as the CI (1.04 for wavefront data). By corneal values, however, the FE (1.76 D) was approximately 10% less than the SIA (1.97 D) because the treatment was off-axis (counterclockwise; [CCW]) by 13 degrees (AE). Thus, a loss of effect at the intended (53-degree) meridian occurred and was compounded by the undercorrection. This is highlighted by a reduction in the FI (0.59) as compared to the CI (0.67) for topography and a significant proportion of the SIA being ineffective as CW torque (0.88 D). The CW rotation of the existing corneal astigmatism

Table 1. *Individual Astigmatism Analysis*

Measure	Refraction (D)	Topography (D)
Astigmatism (simple subtraction)	–3.12	–0.20
Polar change	1.02	1.34
Flattening effect	3.09	1.76
Flattening index	1.04	0.59
Coefficient of adjustment	0.96	1.50
Torque effect (CW)	0.18	0.88
Ocular residual astigmatism (ORA)	1.53 D × 152	

CW = clockwise

meridian from 134 to 93 degrees would have been greater and closer to the targeted value of 1.53 D @ 62 if the full TIA was achieved by having the SIA on axis and if the CI was closer to 1.0.

The flattening and torque effects of the SIA employing a topographical analysis are displayed in Figure 18. This analytical approach can be applied independently to any mode of astigmatism measurement. Where axis differences exist between corneal shape and refraction, an on-axis correction applied to one will necessarily have an off-axis effect on the other. Simple subtraction analysis of preoperative from postoperative astigmatism shows a significantly greater decrease in astigmatism refractively (–3.12 D) as compared to corneal astigmatism (–0.20; see Table 1). Vector planning and calculation of the topographical target shows that this low level of corneal correction was expected when the treatment emphasis was exclusively directed at correcting refractive astigmatism. Corneal outcomes could have been improved by including corneal preoperative parameters in planning.

Polar analysis reveals some WTR net astigmatic change in both modes of measurement. This is consistent with the orientation of the SIA (steepening) at axes of 55 degrees refractively and 66 degrees for topography (i.e., within a 45- to 135-degree range).

The amount of with-the-rule or against-the-rule change induced on the existing astigmatism can be calculated by using 90 degrees as the reference axis (Fig 19).[9] An ORA of 1.53 D × 152 (see Table 1) indicates fairly poor correlation between preoperative refractive (–2.97 D Ax 53 at the corneal plane) and corneal (1.60 D @ 134) astigmatism readings. The difference in magnitude at the corneal plane is large, at 1.37 D, with the angular separation being 9 degrees.

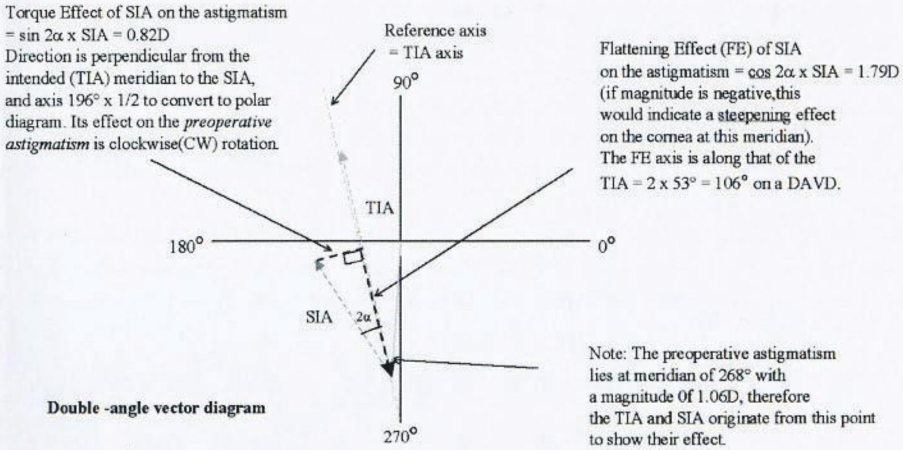


Figure 18. *Alpins method of determining the flattening and torque effects of surgically induced astigmatism at the intended meridian target-induced astigmatism, which is 53 degrees (shown at 106 degrees on the double-angle vector diagram above).*

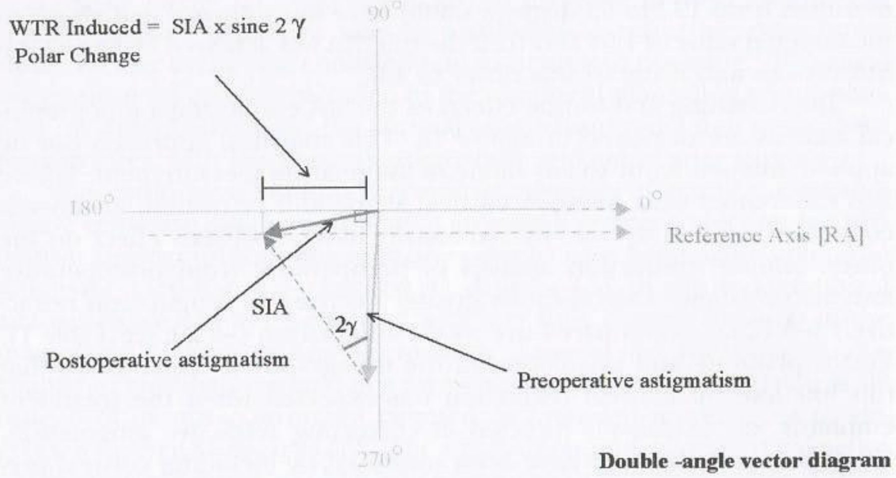


Figure 19. *Polar change induced on existing astigmatism by achieved treatment: double-angle vector diagram. A positive value indicates a net increase in astigmatism at the polar 90-degree meridian (WTR change), and a negative value indicates a net decrease (ATR change).*

When an analogous spherical analysis is performed at the corneal plane (Table 2), the results show an overcorrection of spherical treatment by 34% (S.CI = 1.34), with the remaining spherical equivalent being hyperopic at 1.26DS.

Aggregate Data Analysis When examining astigmatic outcomes of an individual as well as aggregate data, it is also valuable to look at all the modes used to measure astigmatism. This yields a more precise examination of the differing treatment trends revealed by these parameters. Analyses derived from manifest refractive astigmatism values, where the observer is aware that the target result is plano, have a potential for observer bias. However, the use of conventional keratometry or corneal topography provides an objective means of measurement using a device to detect any trends. This advantage of using a device for measurement is also present with wavefront analysis.

Aggregate data analysis of vectors should involve examination of arithmetic means, used to determine the mean vector magnitude where orientation of the vector is not considered. Their ratios and differences can

Table 2. *Individual Analogous Spherical Analysis*

Measure	Refraction
Spherical correction index (S.CI)	1.34
Spherical difference (S.Diff) in diopters	1.26
Spherical index of success (S.IOS)	0.34

also be examined. Examination should also use vector means, a head-to-tail summation of vectors incorporating their orientation as well as magnitudes. The vector mean is always less than the arithmetic mean magnitude, and the greater the difference, the less any overall trend is evident. Multiple eyes can be examined in a number of ways:

The **arithmetic means** of vector magnitudes can be calculated (without reference to their respective axes). These are effectively displayed alongside the vector mean values and the polar surgical vector graphs, such as those shown in Figure 20, which display pictorially the change that is happening as it relates to the eye.

The TIA arithmetic mean is the average astigmatic treatment at the corneal plane applied to the eyes. The SIA arithmetic mean is used to determine the average overall astigmatic change and, when compared to the arithmetical mean, TIA can show overall over- or undercorrection. The DV arithmetic mean is a precise averaged induced astigmatic change that would enable the initial surgeries to achieve their intended targets—an absolute measure of success.

The **arithmetic ratios** between each of the SIA, DV, and the FE to the TIA vector magnitudes can be compared for a more detailed analysis. Each ratio provides separate information necessary for understanding the clinical relevance of any astigmatic change.

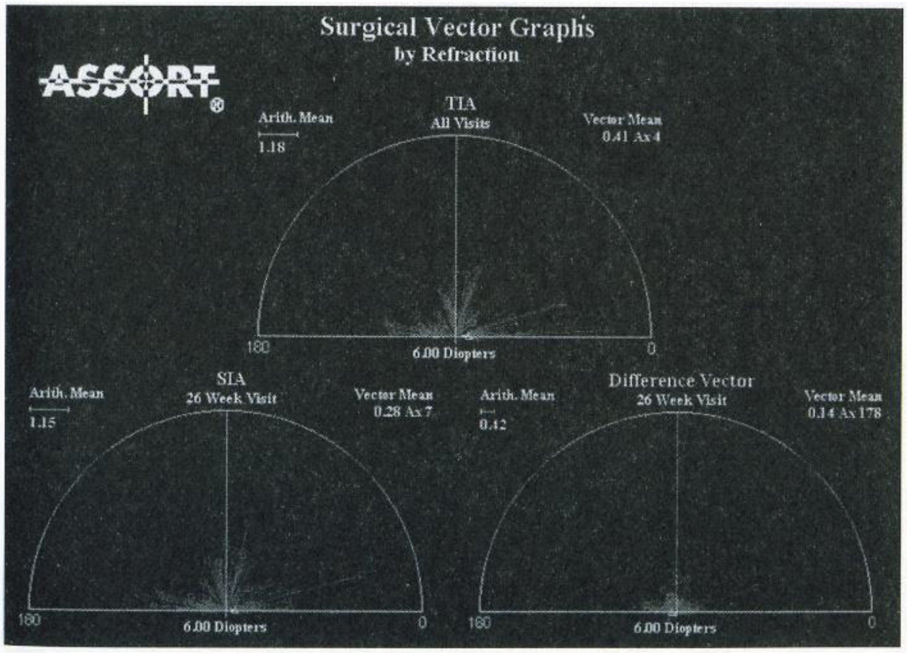


Figure 20. *Surgical vector graphs display each individual treatment at the meridian of maximum ablation (TIA axis).*

Coefficient of Adjustment The coefficient of adjustment (CA) is calculated by dividing the TIA by the SIA. This is the adjustment required to modify future astigmatism treatment magnitudes (TIA) to improve outcomes. It is the inverse of the CI value and is preferably 1.0. A value greater than 1 indicates undercorrection, and less than 1 reveals an overcorrection of astigmatism magnitude.

The CI and the CA can be averaged for a series of eyes by calculating their geometrical means. This is derived by taking the mean of the individual logarithmic values followed by the antilog of this calculated mean[2]. If the geometrical mean varies significantly from unity, a trend is apparent. The IOS geometrical mean is calculated by taking a mean of the individual square root values, then squaring this calculated mean value.

Flattening Effect The flattenning effect (FE) is the amount of astigmatism reduction achieved by the effective proportion of the SIA at the intended meridian: FE = SIA cos. 2 (AE) (see Fig 18). As the AE increases (which is simply the angle separating the SIA from the TIA on a polar diagram), the amount of FE decreases, resulting in increasing torque; that is the ineffective part of the SIA that is not reducing astigmatism but is rotating it. Calculating its arithmetic or aggregate mean is of limited interest because the FE varies according to each differing amount of astigmatic treatment (TIA). Its value lies in examining individual patient outcomes.

Flattening Index The FI is calculated by dividing the FE by the TIA and is preferably 1.0. The FI cannot exceed the CI for any one eye, as can be seen by the formula:

$$\mathrm{FI} = \frac{\mathrm{SIA} \times \cos 2\,(\mathrm{AE})}{\mathrm{TIA}}$$

When SIA = TIA, a full correction of astigmatism is achieved, resulting in a CI of unity. The FI becomes smaller as a function of increasingly misaligned treatment and is 0 at 45 degrees off-axis.

The **arithmetical differences** between SIA and TIA axes can be compared. The *magnitude of error* (ME) is the arithmetic difference between the magnitudes of the SIA and TIA. The ME is positive for overcorrections and negative for undercorrections. The angle of error (AE) is the angle described by the vectors of the achieved correction (SIA) versus the intended correction (TIA). The AE is positive if the achieved correction is on an axis CCW to where it was intended and negative if the achieved correction is CW to its intended axis. It is not useful to average axes of astigmatism or vector values themselves but only their separation—the AE.

However, by plotting a scatter graph of the AE (Fig 21), we can determine whether there is a bias to any one coincident value. Figure 21A shows a group of 100 eyes analyzed 12 weeks postoperatively (LASIK) by refraction

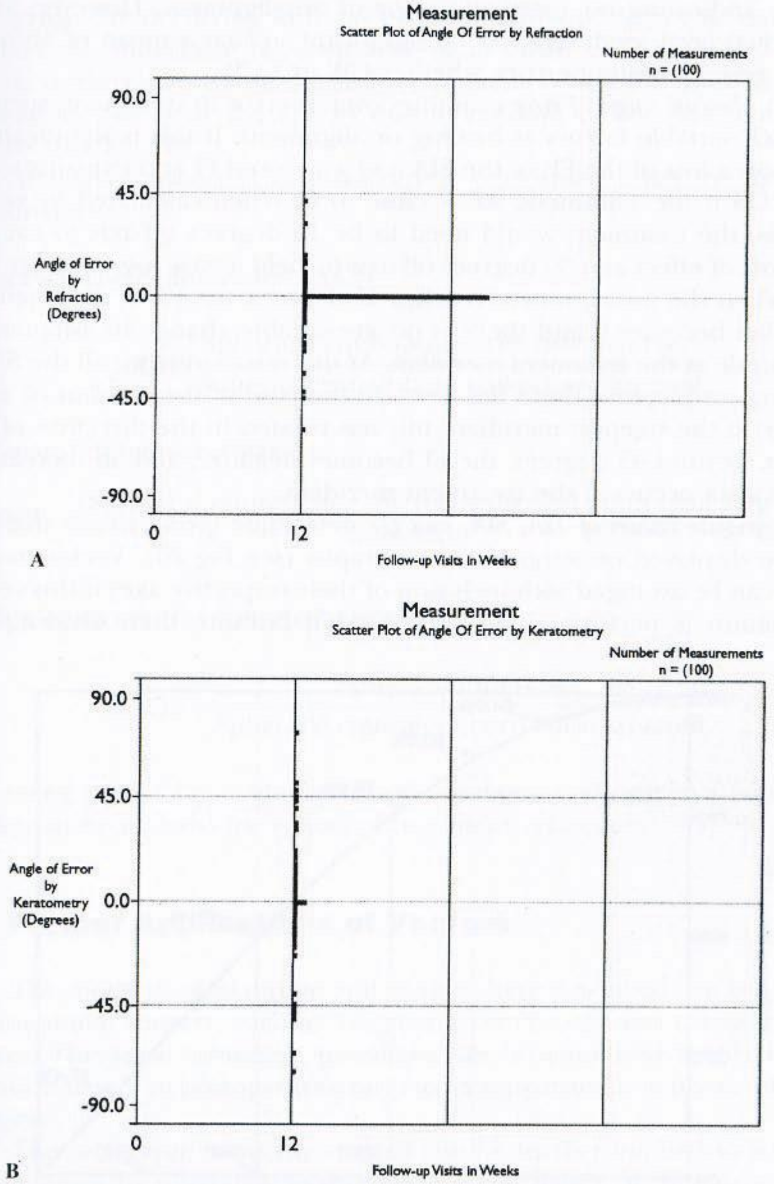


Figure 21. *(A) The angle of error by refraction value shows the amount each treatment applied was off-axis. A significant proportion of these occurs at 0 degrees suggesting some measurement bias. (B) The angle of error by corneal values shows the amount by which each treatment applied was off-axis. In this example, there is no bias to any one coincident value.*

and indicates a bias to an AE of 0 degrees. Figure 21B shows the same group of 100 patients analyzed by keratometry; the measurements in this example show no bias to any one value. The mean AE may be minimal in a group analysis (due to positive and negative angles cancelling each other), indicating no systematic error of misalignment. However, at an individual level, each AE may be significant so that a mean of absolute values reveals existing errors, whether CW or CCW.

An *absolute angle of error* examines overall error in treatment, suggesting such variable factors as healing or alignment. If this is significant, it will show a loss of the FE of the SIA and a reduced FI and explains well a high IOS if the arithmetic AE is close to 0. When calculated by vector analysis, the treatment would need to be 15 degrees off-axis to cause a 15% loss of effect and 30 degrees off-axis to yield a 50% loss of effect (Fig 22). When the misalignment reaches 45 degrees, the FE at the intended meridian becomes 0 and there is no measurable change in astigmatism magnitude **at the treatment meridian.** At this misalignment, all the SIA is creating torque, and there has been an increase in the amount of astigmatism at the steepest meridian; this has rotated in the direction of the torque. Beyond 45 degrees, the FI becomes negative, and an increase in astigmatism occurs at the treatment meridian.

Aggregate values of TIA, SIA, and DV determine group trends that can also be displayed on surgical vector graphs (see Fig 20). Vector magnitudes can be averaged with inclusion of their respective axes if this vector summation is performed in a head-to-tail fashion, then dividing the

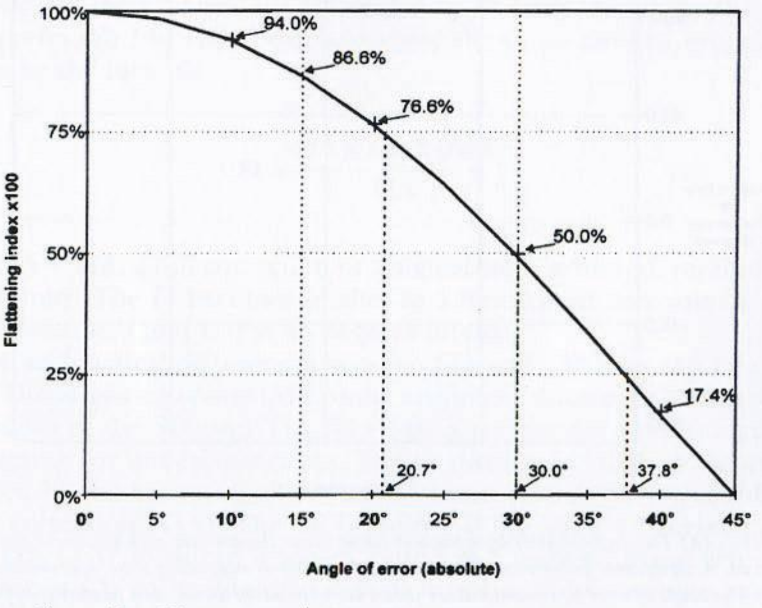


Figure 22. *Effect of misaligned astigmatism treatment on flattening index.*

net resultant vector's total length by the number of its individual components. The summated vectorial mean enables a trend analysis of aggregate data. For example, the DV vectorial mean will be less than the arithmetic magnitude mean of DVs and is useful in determining whether any systematic errors are occurring with any group of patients' aggregate data. The greater the difference between arithmetic mean and summated vector mean, or the closer the summated vector mean is to 0, the more likely the magnitude mean of the DVs is due to random rather than systematic errors (see Fig 20).

Analogous spherical analyses can be performed using the following parameters:

Spherical correction index (S.CI):

$$\text{S.CI} = \frac{\text{Spherical equivalent correction achieved}}{\text{Spherical equaivalent correction targeted}}$$

Spherical difference (SDiff):

$$\text{SDiff} = \frac{[\text{Spherical equivalent achieved–spherical equivalent targeted}]}{(\text{absolute})}$$

Index of success for spherical change (S.IOS):

$$\text{S.IOS} = \frac{\text{Spherical difference}}{\text{Spherical equivalent correction targeted}}$$

Inverting the S.CI provides spherical nomogram adjustments (similar to astigmatism analysis) for spherical treatment refinement.

■ Further Applications of Vectors

The methods of planning and analysis here described can be applied to incisional surgery, such as astigmatic keratotomy and cataract procedures. The same treatment paradigm can be employed by placing the incision based on postoperative targeted astigmatism as in photorefractive surgery.

The technique of vector analysis can be further applied to irregular astigmatism by separately examining the two halves of the cornea. This requires adding a second analysis between 181 and 360 degrees displaying both together on a 360 degree polar diagram as they would appear on the eye and on a 720 degree DAVD for the process of vector analysis and target determination. Manifest refraction provides only one refractive astigmatic value applicable to both sides of the cornea. However, wavefront refraction, like corneal topography, can provide two fundamentally important values and many more. The method and principles of vector planning have been described in detail in a previous paper.[11]

■ Conclusion

The application of vectors in planning for and treating and analyzing photoastigmatic refractive surgery allows for improvement in patient outcomes in various ways. *Incorporating both the topography and refractive values into the surgical plan* potentially reduces the amount of corneal astigmatism (second- and third-order aberrations) remaining.[13] This orientation of the remaining astigmatism is biased to a favorable with-the-rule position. *Calculating the TIA can achieve the maximum correction of astigmatism* to minimize the total amount of refractive and corneal astigmatism remaining to the minimum, as calculated by the magnitude of the ORA.

Determining where the targets and their orientation lie provides guidance for the apportionment of the remaining astigmatism between the cornea and refraction. *Analyzing individual outcomes* provides a comprehensive understanding of any induced astigmatic change, enabling an integrated and valid examination of these changes that are applicable to keratometry, topography, manifest refraction, or wavefront refraction. *Periodically analyzing aggregate data* using this method of astigmatism analysis can determine success and identify errors for the independent refinement of both spherical and astigmatic nomograms.

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