Vector analysis of astigmatism changes by flattening, steepening, and torque

Noel A. Alpins, FRACO, FRCOphth, FACS

ABSTRACT

Purpose: To understand the effect of astigmatism surgery by analyzing astigmatic changes according to their component parts.

Setting: Cheltenham Eye Centre, Melbourne, Australia.

Methods: The component parts of the astigmatic changes considered were flattening, steepening, clockwise torque, and counterclockwise torque. Calculations to determine the astigmatic change were performed by vector analysis using rectangular coordinates after doubling the astigmatism and surgical vector axes. A reference axis was used for the resolution of astigmatic change to ascertain its effect along a selected meridian.

Results: When correcting astigmatism, the orientations of incisional (tissue addition) or nonincisional (tissue ablation) techniques in any cornea are at right angles to each other. Since differences exist in the measured astigmatism depending on whether it is measured topographically or by manifest refraction, an on-axis correction in one will occur with an off-axis effect in the other. The net result is a reduced flattening effect and a proportionately increased torque effect for the off-axis component.

Conclusion: When treatment is applied off one of the four primary axes, the treating vector can be resolved into its component parts of flattening, steepening, and torque. Analyzing changes in this way provides a uniform means of assessing astigmatic changes for all types of cataract and refractive surgery and quantifies the flattening effect when treatment is applied off the intended meridian. *J Cataract Refract Surg 1997; 23:1503–1514*

The general goal of incisional or ablative astigmatic surgery is to reduce the magnitude of astigmatism by flattening the cornea at its steepest meridian, steepening the cornea at its flattest meridian, or a combination. The multiple factors at play may lead to significant variations between the intended or targeted axis and the achieved or induced axis of treatment.

It is useful, therefore, to have an analytical technique that addresses any imprecision in the attempt to correct astigmatism and that allows the determination of the actual achieved flattening at the desired axis. The force that induces a change in the shape of the cornea can be resolved into two component parts: (1) one that will lie on the intended axis and will thus have some flattening (or steepening) effect; (2) one, referred to as clockwise or counterclockwise torque, that is ineffective in reducing existing corneal astigmatism and that lies at 45 degrees to that intended axis.

Presented in part at the Symposium on Cataract, IOL and Refractive Surgery, San Diego, California, USA, April 1995.

The author has a financial interest in the subject matter.

Reprint requests to Noel A. Alpins, FRACO, FRCOphth, FACS, Cheltenham Eye Centre, 7 Chesterville Road, Cheltenham, Melbourne, Victoria 3192, Australia.

Analysis techniques using polar astigmatism values have addressed the steepening or flattening effect of cataract surgery and intraocular lens implantation[1,2] at the 0, 90, and 180 degree polar axes. However, these techniques have limitations when analyzing incisions away from these polar axes.[3,4]

The method of Holladay and coauthors[3] uses only polar coordinates to compute with-the-wound and against-the-wound change. Supplementary calculations are required, and 180 degree adjustments must be made to determine the axis of the change.[5] Similar adjustments are required with Naeser and coauthor's[4] modified formula, which also uses polar coordinates to determine what they call with-the-power and against-the-power change.

These terms, which may be appropriate for incisional surgery, do not readily refer to cases in which corneal tissue is ablated or contracted at a meridian 90 degrees away. A simplified manner for universally addressing astigmatic change at the intended axis is effectively addressed by the terms *flattening* and *steepening* for all incisional and nonincisional refractive techniques.

This paper introduces two formulas that allow quantification of the flattening, steepening, and torque components. It also presents vector diagrams using rectangular coordinates to show the methodology for determining the effect of an astigmatism treatment at "off-axis" orientations. The accurate calculation of the effective flattening occurring at *any* reference meridian is enabled by resolution of the vector forces. The example of refractive cataract surgery will be used to illustrate these effects.

Also provided is a graphic that shows the diminishing flattening effect of an astigmatism treatment as its amount of deviation increases from the intended axis. The amount of astigmatism that remains, and the induced shift in orientation of the existing astigmatism that occurs as a result of this increasing off-axis effect, is also displayed.

Theory and Methods

The ASSORT refractive surgery planning and outcomes analysis computer program (run on an IBM-compatible 80486DX PC with 8 MB of RAM) was used to calculate all parameters. In Figures 1 to 6, these values are shown in the boxes that appear below the figures.

Figures 1 to 6 illustrate the concepts of flattening, steepening, and torque. (The legend for these figures and the equations in this paper is in the Appendix.) The astigmatism and surgical vector diagrams, as in Figure 1A, use polar coordinates that represent these values as they would be measured on, or applied to, the cornea. The double-angle vector diagrams (DAVD), as in Figure 1B, use rectangular coordinates after doubling astigmatism and surgical vector axes; however, their magnitudes are unchanged.

The preoperative, target, and achieved astigmatism values refer to astigmatism measurements in general. In this example, corneal astigmatism is discussed; the values represent the steepest corneal meridians. However, they could also represent refractive astigmatism values at the corneal plane as determined by the power axis of the negative cylinder, which is equivalent to the cylinder axis of the positive cylinder. The corneal and refractive (corneal plane) astigmatism values are separately examined in Figure 6. These relative changes in corneal curvature usually occur concurrently with generalized changes in corneal curvature associated with spherical refractive error treatment.

Flattening, Steepening, and Torque: Basic Concepts

The example in Figure 1A shows a cornea for which the preoperative astigmatism value has been measured at 1.40 diopters (D) at a meridian of 8 degrees. This is also shown in Figure 1B on the DAVD, where the orientation is 16 degrees. The meridian of this preoperative astigmatism is chosen as the reference axis. This reference axis is the same for any one treatment whether it is performed by incisional or ablation surgery, where the meridians of greatest treatment activity to produce the same astigmatism effect lie at right angles to each other.

The concept of steepening refers to the only effect induced by a vector (steepening) force that, when applied to the steepest meridian of the cornea, leads to increased astigmatism at that treated meridian. Conversely, flattening refers to the only effect induced by a vector (steepening) force that, when applied at 90 degrees to the steepest meridian of the cornea, would lead to reduced corneal astigmatism.

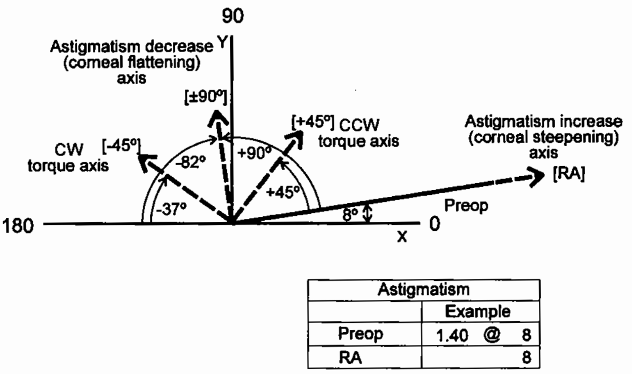


Figure 1A. (Alpins) Flattening, steepening, and torque principal meridians: astigmatism and surgical vector diagram.

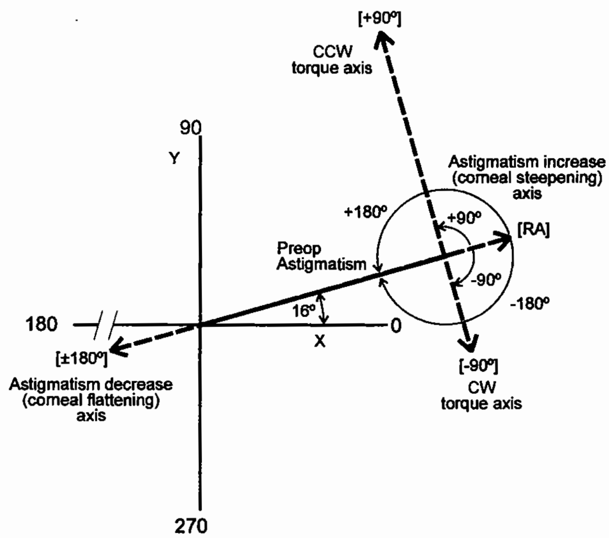


Figure 1B. (Alpins) Flattening, steepening, and torque principal meridians: double-angle vector diagram.

Determining the flattening and steepening effects of on-axis treatment. It is a simple arithmetic process to calculate that any treatment (referred to as a steepening force such as the surgically induced astigmatism [SIA] or target induced astigmatism [TIA]) placed at that reference axis will increase astigmatism. If the treatment is placed 90 degrees away on the cornea (Figure 1A), it will reduce astigmatism. (This is the equivalent of 180 degrees away as seen on a DAVD, as in Figure 1B.) If the opposing TIA is greater than the preoperative astigmatism, it will leave astigmatism at the opposite axis by an amount equal to the arithmetic difference of the two values.

Astigmatism torque occurs as the only effect induced by a vector (steepening) force that, when applied at 45 degrees to the steepest meridian of the cornea, induces no change to the magnitude of the astigmatism measurable at this steepest meridian. Rather, it leads to a change in the magnitude of the existing astigmatism and an induced shift in its orientation toward that of the treating vector force.

Figures 1A and 1B, respectively, show a torque force at 45 and 90 degrees counterclockwise (CCW) to the reference axis that is aligned with the existing astigmatism. The respective opposing torque forces are at −45 and −90 degrees clockwise (CW) to this meridian.

In summary, when the astigmatic treatment vector is aligned with the existing astigmatism, relative steepening occurs and the astigmatism is increased. When the astigmatic treatment vector is perpendicular to the meridian of existing astigmatism, relative flattening occurs and the astigmatism is decreased. When the astigmatic treatment is aligned at 45 degrees to the existing astigmatism, a CW or CCW torque induces no change to the magnitude of the astigmatism measurable at the preoperative (reference) meridian but alters the magnitude and orientation of the existing astigmatism.

Treatment Applied Oblique to the Reference Axis

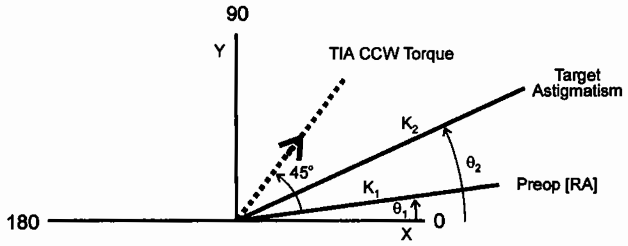
Determining magnitude and axis of astigmatic torque. Any force applied to the cornea will exert a purely torque effect if it is oriented obliquely (at 45 degrees) to the reference axis (Figure 2A) and will, therefore, have no flattening or steepening at that reference axis. The effect of this force on astigmatism can be seen by examining the DAVD in Figure 2B, which illustrates that the target astigmatism increases in magnitude and its orientation shifts farther from the preoperative meridian as the torque force increases.

In this DAVD, this tangential vector force lies perpendicular to the preoperative astigmatism, indicating that no flattening or steepening effect has occurred at the reference axis on the preoperative astigmatism meridian.

Figure 2B shows the following:

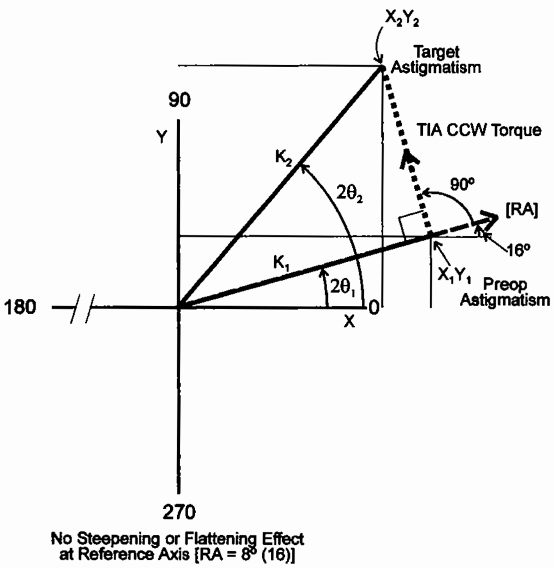
$$X_1 = K_1 \text{ cosine } (2\theta_1)$$
$$Y_1 = K_1 \text{ sine } (2\theta_1)$$
$$X_2 = K_2 \text{ cosine } (2\theta_2)$$
$$Y_2 = K_2 \text{ sine } (2\theta_2)$$

The vector axis (double angle) and the magnitude as determined previously[6] are represented by



Astigmatism		
		Example
Preop	K_1 Ax θ_1	1.40 @ 8
Targeted	K_2 Ax θ_2	1.69 @ 25

Figure 2A. (Alpins) Determining astigmatic torque: magnitude and axis astigmatism and surgical vector diagram.



Surgical Vectors	
	Example
TIA CCW Torque	0.95 Ax 53

Figure 2B. (Alpins) Determining astigmatic torque: double-angle vector diagram.

$$\text{TIA axis}_d = \arctan \frac{Y_2 - Y_1}{X_2 - X_1}$$

$$\text{TIA magnitude} = \frac{Y_2 - Y_1}{\text{sine (TIA axis}_d)}$$

The magnitude of the astigmatism torque force is determined as:

$$K_{\text{Torque}} = K_2 \text{ sine } (2\theta_2 - 2\theta_1)$$

If the result is positive, the force is in a CCW direction and if the result is negative, it is CW.

The amount of steepening that has occurred at the preoperative astigmatism axis is represented as

$$K_{\text{Steepening}} = K_2 \text{ cosine } (2\theta_2 - 2\theta_1) - K_1$$

If the result is zero, it indicates that no flattening or steepening effect has occurred. If the value is positive, steepening has occurred and if it is negative, it would indicate a flattening effect.

Figure 2A shows a treatment applied to the cornea 45 degrees from the preoperative astigmatism meridian; the resulting astigmatism has rotated and has a different magnitude than its preoperative value. To rotate this astigmatism—1.40 D Ax 8 degrees (or 16 degrees on Figure 2B, a DAVD)—to an arbitrarily chosen meridian of 25 degrees (50 degrees), the direction of the torque force that must be applied is CCW and can then be plotted on Figure 2B at 90 degrees to the existing astigmatism, thus orientated at axis 106 degrees.

The intersection of the lines joining the TIA CCW torque vector force with the target meridian determines the magnitudes of the target astigmatism (K_2) and CCW torque.

$$K_1 = K_2 * \text{cosine } (2\theta_2 - 2\theta_1)$$

$$\text{Target Astigmatism } K_2 = \frac{1.40}{\text{cosine } (50 - 16)}$$

$$= 1.69 \text{ D}$$

$$K_{\text{TIA}} \text{ Torque} = K_2 * \text{sine } (2\theta_2 - 2\theta_1)$$

$$= 1.69 * \text{sine } (50 - 16)$$

$$\text{CCW Torque} = 0.95 \text{ D}$$

The TIA CCW torque vector is shown at axis 106 degrees (16 + 90 degrees) on the DAVD by placing its tail at the head of the preoperative astigmatism vector. It is then displayed on the surgical vector diagram by halving its axis value and locating its tail at the zero point on the polar diagram.

Treatment Applied Off Axis to the Principal Meridians

Two separate vector forces are displayed on the DAVD, shown in Figures 3A and 3B. Each force can be resolved into its two component parts, one of which projects onto the reference axis. This allows determination of the net effect of treatment at the reference axis when the treatment is applied at another orientation.

Both figures show that the treatment is off axis by 2α degrees, where α represents the angle subtended between the intended treatment and the reference axis, in this case the meridian of the preoperative astigmatism at 8 degrees (16 degrees). Figure 3A shows the net

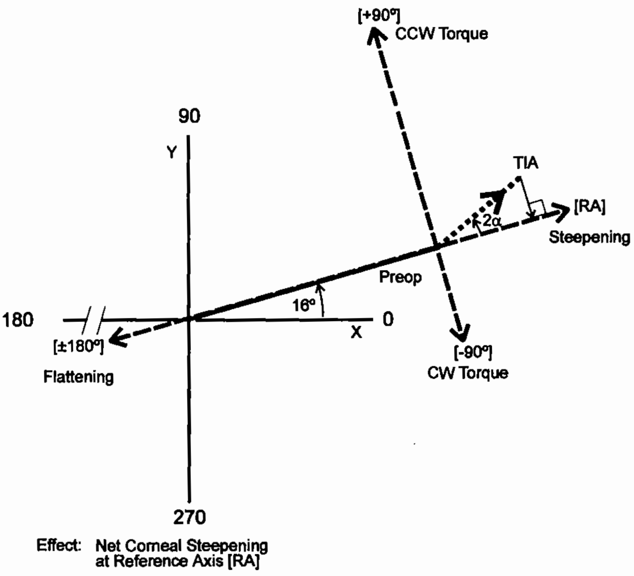


Figure 3A. (Alpins) Determining flattening or steepening effect of treatment: double-angle vector diagram.

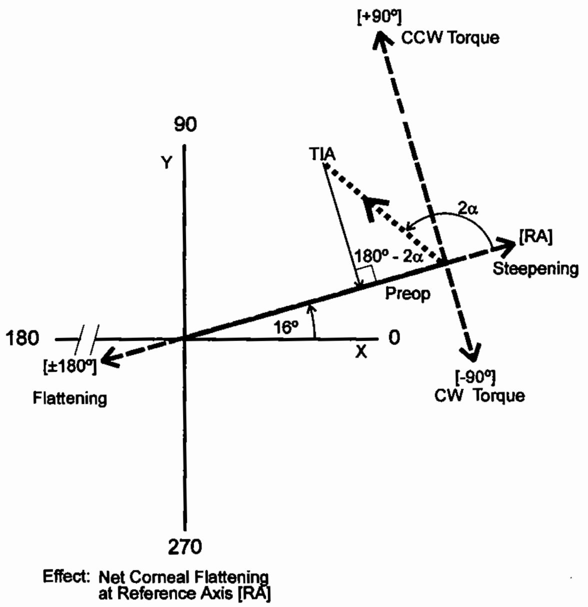


Figure 3B. (Alpins) Determining flattening or steepening effect of treatment: double-angle vector diagram.

steepening and Figure 3B, net flattening at the reference axis.

Determining the steepening effect (Figure 3A). Simple trigonometry can be used to show that the applied vector force has a component that acts along the reference axis and will, therefore, affect the existing astigmatism. The magnitude of that component can be calculated as TIA cosine 2α (adjacent = hypotenuse × cosine angle subtended). Thus, the net steepening effect is TIA cosine 2α.

Determining the flattening effect (Figure 3B). Flattening occurs when cosine 2α is negative, which occurs when α is greater than 45 degrees or less than −45 degrees or 2α is greater than 90 dgrees or less than −90 degrees. The net steepening effect is TIA cosine 2α, which is equivalent to −TIA cosine (180 degrees − 2α).

The net flattening effect is another way of describing a negative net steepening effect.

A negative value is obtained for cosines in the range of 90 to 270 degrees or −90 to −270 degrees. This is consistent with the diagrammatic findings of Figure 3B, which shows that a flattening effect, or a reduction in astigmatism, has been achieved in the range of 90 degrees, either side of the 180 degrees axis that is opposite to the reference axis.

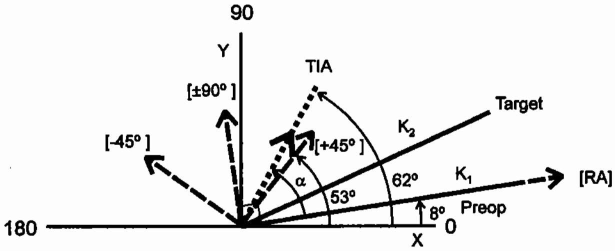
Any surgical procedure performed to reduce astigmatism within 45 degrees of its intended placement will have some flattening and torque effect on the cornea. This flattening effect decreases as a function of the cosine of twice its increasing angular misplacement. A corresponding increase occurs in the torque effect, which will be discussed in more detail later.

Changing the Astigmatism Meridian Without a Change in Magnitude

In the next example, the surgical intention is to change the orientation of the existing astigmatism from 8 degrees to an arbitrarily chosen meridian of 25 degrees (as shown in Figure 2); however, in this case the magnitude is left unchanged at the preoperative value of 1.40 D.

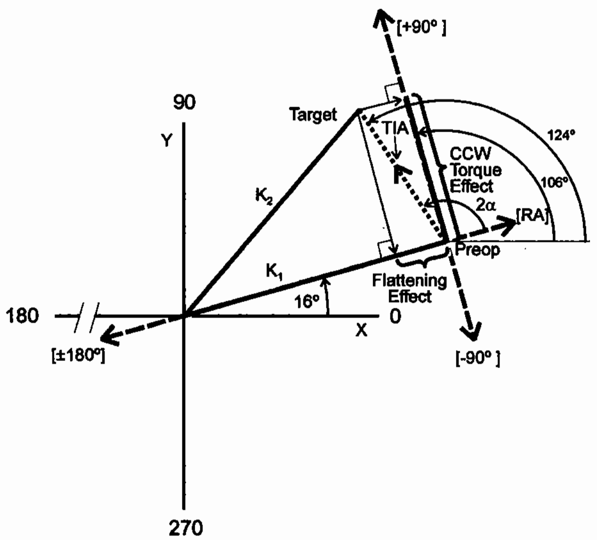
The magnitude and orientation of the required astigmatism treatment vector are shown in Figure 4B by the dotted line representing the tail, connecting the preoperative value to its head at the target astigmatism value. It is displayed on the surgical vector diagram (Figure 4A) by halving its axis value and locating its tail at the zero point. The steepening force required to perform this task does not coincide with the CCW torque axis at 53 degrees but lies a further 9 degrees CCW to it at 62 degrees.

Unlike the purely CCW torque, the TIA in Figure 2, this treatment has two components to its effect. Each component can be separately quantified by projecting the vector onto the reference (preoperative) axis for its flattening effect or the axis at 90 degrees CCW to the reference axis for its torque effect.



Astigmatism	
	Example
Preop	1.40 @ 8
Targeted	1.40 @ 25
α	54°

Figure 4A. (Alpins) Determining separate flattening and torque effects when changing the astigmatism meridian without a change in magnitude: astigmatism and surgical vector diagram.



Surgical Vectors	
	Example
TIA	0.82 Ax 62
CCW Torque Effect	0.78 Ax 53
Flattening Effect	0.25 Ax 8
α	54°

Figure 4B. (Alpins) Determining separate flattening and torque effects when changing the astigmatism meridian without a change in magnitude: double-angle vector diagram.

Changing orientation without a change in the magnitude of the astigmatism requires some flattening in combination with torque. The greater the change in orientation in the existing astigmatism, the greater the proportion of the flattening component and the less the torque. At the maximum change of astigmatism meridian, which is 90 degrees (180 degrees on DAVD), the treatment required is wholly a flattening one and the torque component becomes nil.

Applying the principles in practice. It is useful to divide any change in corneal astigmatism into its two component parts: flattening (or steepening) and torque. In general, the greater the proportion of torque effect, the less effective the treatment in reducing astigmatism.

This is also a consideration when analyzing refractive surgery outcomes as differences exist between topographic and refractive astigmatism. A treatment placed on axis for flattening of refraction will be off axis for topography and vice versa.

Furthermore, a treatment will have some torque effect on both modalities when it may not be aligned with either the topographic or the refractive axis but be between the two, for example when treating optimally.[7] Therefore, the treatment in these cases will have a torque as well as a flattening effect on the astigmatism.

In Figure 4A, α represents the amount the TIA is off axis from the reference axis. A steepening force acting at the reference axis would act entirely to increase the existing astigmatism. The torque is determined as

$$K_{\text{Torque}} = \text{TIA sine } 2\alpha$$
$$= 0.82 * \text{sine } (2 * 54)$$
$$= +0.78 \text{ (the positive value indicates the force to be CCW to the reference axis [8 degrees])}$$
$$K_{\text{Torque}} \text{ Axis} = 8 + 45 = 53$$

The amount of flattening or steepening is determined as

$$K_{\text{Flattening/Steepening}} = \text{TIA cosine } 2\alpha$$
$$= 0.82 * \text{cosine } (2 * 54)$$
$$= -0.25 \text{ (the negative value indicates the effect to be flattening at the reference axis [8 degrees])}$$

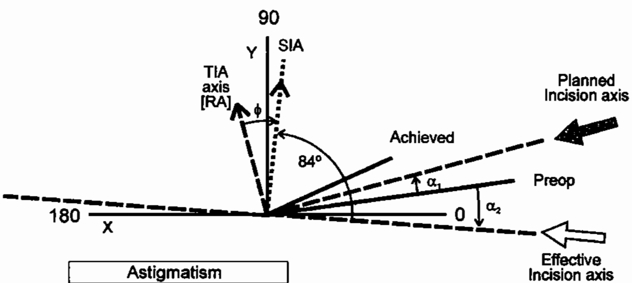
There is no simple arithmetic relationship between the induced change (TIA) and its two component parts; however, they are related by their squared values as together the three form the sides of a right-angled triangle and so are related by Pythagorean theorem

$$(K_{\text{TIA}})^2 = (K_{\text{Flattening/Steepening}})^2 + (K_{\text{Torque}})^2$$

Incisional Refractive Surgery: Determining the Effect of a Planned Off-Axis Treatment at That Axis

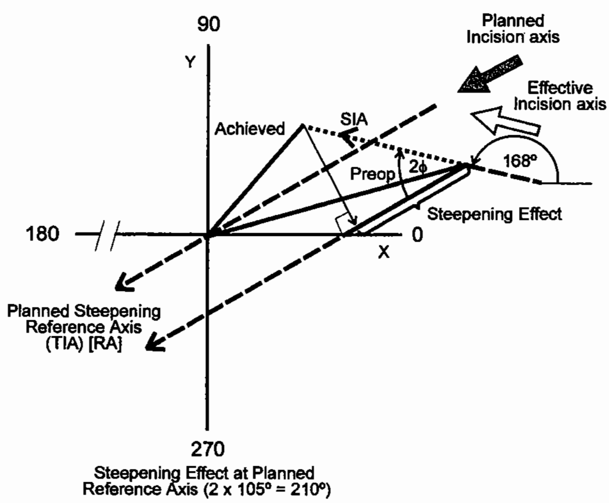
Using the planned surgical treatment axis as the reference axis. The reference axis may be the preoperative meridian, as in Figures 1 to 4. However, it may also be any other orientation on the cornea. This more complicated mode of analysis is necessary as the planned treatment (which may be an incision or an ablation) may not always coincide with the preoperative astigmatism. This would be the case in the following:

1. When cataract surgery using a temporal incision is performed but the steepest corneal meridian is orientated elsewhere.



Astigmatism	
	Example
Preop	1.40 @ 8
Incision	@ 15
Achieved	0.75 @ 25
α_1	7°
α_2	-14°
ϕ	-21°
RA	105

Figure 5A. (Alpins) Incisional refractive surgery; effect of planned "off-axis" treatment: astigmatism and surgical vector diagram.



Surgical Vectors	
	Example
TIA	Ax 105
Achieved Steepening SIA	0.88 Ax 84
ϕ	-21°
Steepening Effect	0.65 Ax 105

Figure 5B. (Alpins) Incisional refractive surgery; effective steepening of planned treatment: double-angle vector diagram.

2. In excimer laser surgery for astigmatism planned by refractive astigmatism values alone in which an analysis is performed for the flattening effect of the surgery using topographical values. The steepest topographical meridian usually differs from the refractive axis.[7]

3. When treating astigmatism optimally and the treatment axis does not coincide with either refractive or topographic meridians. In this case, the reference axis will not coincide with the refractive axis or the steepest corneal meridian.

Figures 5A and 5B show how to determine from the measured result of surgery the effect that was achieved at the intended axis when the treatment was planned to be off the steepest corneal meridian and was not effectively applied at its intended orientation. In this example, the reference axis does not coincide with that of the preoperative astigmatism; it lies at the meridian to be steepened, which is 90 degrees from the surgical incision. The example uses the same preoperative astigmatism value of 1.40 D at axis 8 degrees (Figure 5A), displayed at 16 degrees on the DAVD (Figure 5B) as those shown in previous figures.

Determining the functional effect of a treatment. The incision placed at 15 degrees (30 degrees) is 7 degrees (14 degrees) off axis CCW from the meridian of the existing astigmatism (α_1) (Figure 5A). This treatment could be achieved by an astigmatic keratotomy or refractive cataract surgery incision centered at 15 degrees. It could also be performed by an ablation that maximally steepens the cornea at 105 degrees. The axis of the TIA, and in this case the reference axis, is 105 degrees, whether the surgery is done by an incision or ablation.

The measured postoperative astigmatism value achieved in the example is 0.75 D with an axis of 25 degrees, displayed also in Figure 5B at 50 degrees (DAVD). From the preoperative and postoperative values shown in Figure 5B, the SIA vector[6,8,9] is determined to be 0.88 D at axis 168 degrees, which is axis 84 degrees on the polar diagram (Figure 5A). This suggests that the flattening effectively occurred at 174 degrees, 90 degrees from the orientation of the SIA.

In this example the incision was functionally acting as though it were placed at 354 degrees (such as temporally in a left eye), effectively off axis by 14 degrees CW (α_2) from the preoperative astigmatism, and 21 degrees CW from its intended placement at 15 degrees.

The angle ϕ subtended between the intended axis (TIA) and the effective axis (SIA) is −21 degrees and is a measure of the angular amount (CW) the treatment was inaccurately applied taking all surgical variables into consideration. This is the angle of error described earlier.[6]

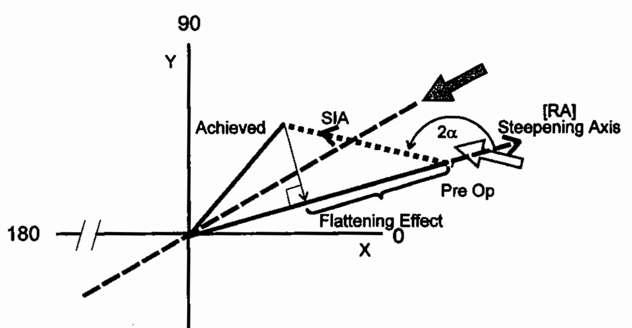
Determining a treatment's steepening effect. Projecting the SIA vector perpendicularly onto a line parallel to the planned incision axis, which is 210 degrees on the DAVD (Figure 5B), determines the amount of steepening achieved at this reference axis. The simple trigonometric calculation SIA * cosine (2 * ϕ) shows the value at axis 210 degrees to be +0.65 D (a positive value), indicating the amount of steepening that has occurred at the intended orientation of 105 degrees.

Determining flattening at the preoperative astigmatism meridian. The amount of flattening at the preoperative astigmatism meridian of 8 degrees (16 degrees) can be derived for this example, shown in Figure 5C, making this orientation the reference axis and projecting the SIA perpendicularly onto it in the same way as shown in Figures 3B and 4B. In this case, a negative value for effect at the reference axis is obtained (−0.78 D Ax 8 degrees), indicating flattening has occurred on the cornea at the meridian of the existing preoperative astigmatism.

The method can also be applied to determine the astigmatic effect of nonincisional surgery at its meridian of placement. The flattening/steepening component of the SIA determined by this means of resolving surgical vectors provides an alternative way to achieve the same magnitude value[5] as that calculated by Holladay and coauthors[3] and Naeser and coauthors[4] for the meridional power of cylindrical lenses and surfaces.

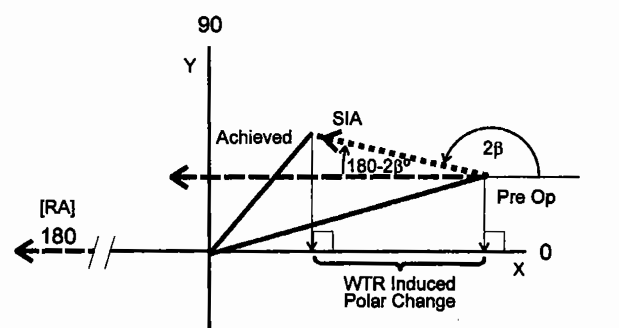
The formula $K_{F/S}$ = SIA cos 2ϕ is directly linked to the SIA and the angle of error, where ϕ is the angular deviation from the treatment's intended meridian of placement. This simplifies the theory and the calculations by eliminating the requirement of previous methods;[3,4] that is, to first determine the meridional power by calculating the contribution of the preoperative and postoperative astigmatism on the surgical meridian and at 90 degrees to it and then subtract one net value from the other to determine the change at that incision's axis. Describing this change as flattening or steepening, according to the corneal change at the reference meridian, should provide some advantages of simplicity and universality for incisional and ablative surgery over the terms with the wound and against the wound[3] and with the power and against the power.[4]

Determining net astigmatism change at the polar axes. The amount of with-the-rule (WTR) or against-the-rule (ATR) change induced on the existing astigmatism can be calculated by using 90 degrees as the reference axis. When β is the axis of the SIA, the magnitude of the polar change is determined by the formula



Surgical Vectors	
	Example
SIA	0.88 Ax 84
α	76°
Preop Axis	@ 8
RA	8
Flattening Effect	0.78 Ax 8

Figure 5C. (Alpins) Incisional refractive surgery; flattening effect on existing astigmatism by achieved treatment: double-angle vector diagram.

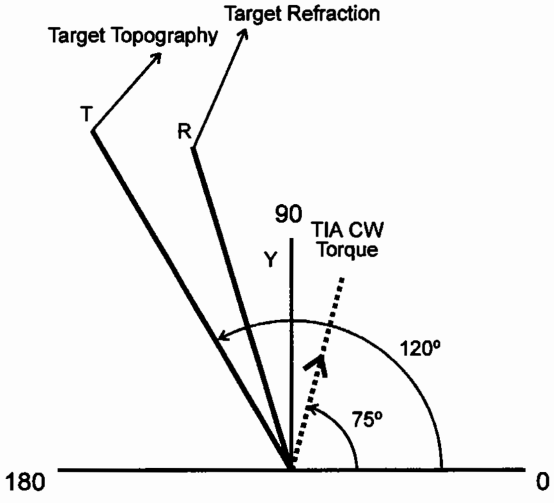


Surgical Vectors	
	Example
SIA	0.88 Ax 84
β	84°
RA	90
WTR Polar Change	0.86

Figure 5D. (Alpins) Incisional refractive surgery; polar change induced on existing astigmatism by achieved treatment: double-angle vector diagram.

$$\begin{aligned}\text{Polar Change} &= \text{SIA} * \text{cosine}\ (180 - 2\beta)\\ &= 0.88\ \text{cosine}\ 12\\ &= 0.86\ \text{D}\end{aligned}$$

A positive value indicates a net increase in astigmatism at the polar 90 degree meridian (WTR change) and a negative value a net decrease (ATR change).



Astigmatism & Surgical Vectors			
	Topography	Plus Cylinder Refraction	Minus Cylinder Refraction
Preop	1.70 @ 120	+1.40 Ax 107	-1.40 Ax 17
TIA		0.83 Ax 75	
Target	1.89 @ 107	+1.93 Ax 96	-1.93 Ax 6

Figure 6A. (Alpins) Integrating topography and refraction; application of an astigmatic torque force to target the preoperative refraction axis: astigmatism and surgical vector diagram.

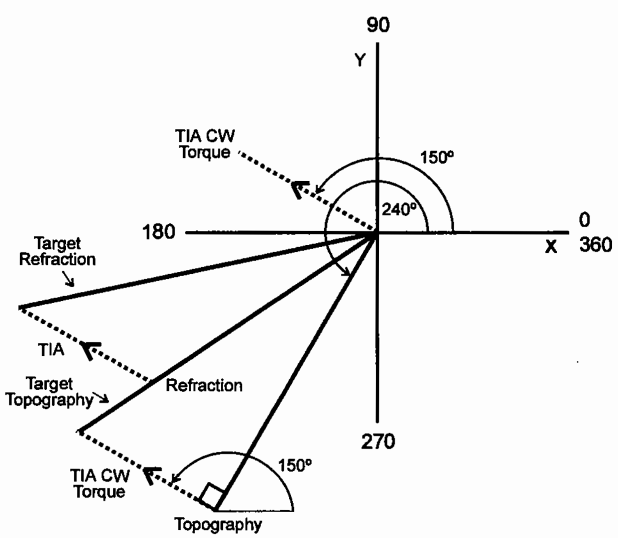


Figure 6B. (Alpins) Integrating topography and refraction; application of an astigmatic torque force to target the preoperative refraction axis: double-angle vector diagram.

Integrating Topography and Refraction

Figures 6A and 6B show the effect of a treatment at a 45 degree angle away from the existing corneal astigmatism and the effect this treatment has on the refractive astigmatism at a different orientation. The first step is to determine the magnitude of this torque treatment vector (TIA) from Figure 6B to bring the topographical axis of 120 degrees (240 degrees) in alignment with a selected axis, such as the preoperative refractive astigmatism axis 107 degrees (214 degrees).

This force lies at axis 75 degrees, 45 degrees CW from the preoperative topography axis at 120 degrees on the polar diagram (Figure 6A). On a DAVD (Figure 6B), this is at a right angle to the topography value (240 degrees) being orientated at 150 degrees. The torque treatment terminates its intersection with the refractive axis at axis 214 degrees. This enables calculation of the target topography magnitude from the intersection at this axis:

$$\text{Preop T} = \text{Target T} * \text{cosine}\ (240 - 214)$$

$$\text{Target Topography} = \frac{1.70}{\text{cosine}\ 26} = 1.89\ \text{D}$$

and the TIA clockwise torque magnitude:

$$\begin{aligned}\text{TIA}_{\text{CW}}\text{Torque} &= \text{Target T} * \text{sine}\ (240 - 214)\\ &= 1.89 * \text{sine}\ 26\\ &= 0.83\ \text{D}\end{aligned}$$

In the next step, the treatment represented by the vector with the same magnitude and orientation, when its tail is applied to the refractive astigmatism value in the DAVD (Figure 6B), will bring it to a new magnitude and axis value, in this case 1.93 D and axis 192 degrees, as shown by joining the head of the treating vector with the graph origin to display calculated target refraction. Even though it is a torque force on the cornea, it is not acting at 45 degrees (90 degrees) to the refractive astigmatism and has some additional steepening effect.

With its axis value halved and locating its tail at the zero point of the polar diagram, the treatment is a CW torque force orientated at 75 degrees, so that the target topography (1.89 D Ax 107 degrees) and refractive astigmatism (+1.93 D Ax 96 degrees) values are both shifted CW from (with larger resultant magnitudes than) their preoperative values. Treatments applied to the corneal astigmatism with a proportion of torque component to their action can bring the separate topographical and refractive axes into a closer or coincident relationship.

Determining Effectiveness of Misaligned Astigmatism Treatment

The flattening effect is reduced as the incision or ablation is displaced from its intended orientation, which is usually that of the preoperative astigmatism (the so-called proper axis).[10] It is a misunderstanding to regard an off-axis treatment as causing an undercorrection of the magnitude of astigmatism.[11,12] When an angle of error exists, it is independent of the magnitude of the SIA and as a result, the proportion of astigmatism corrected as determined by the correction index (which is the ratio of the SIA to the TIA[6,7]) is not affected. However, the result of treatment misalignment is a shift in the orientation of the existing astigmatism toward the axis of the effective steepening treatment—the SIA which in excimer laser surgery is the axis of maximum ablation.

Effective flattening achieved by misaligned astigmatism treatment. In refractive surgery, the pertinent parameter to assess how effective astigmatism treatment has been, is to calculate the amount of flattening effect achieved at the intended meridian of treatment (Figure 5C). This assessment of the effectiveness of any treatment to reduce astigmatism at the intended meridian can be gauged by relating this flattening effect as a proportion of the targeted change in astigmatism—the TIA.

This effective proportion of flattening achieved is the flattening index (FI).

$$FI = \frac{\text{Flattening Effect}}{\text{TIA}}$$

When the surgical change occurs ϕ degrees off the intended axis, the flattening effect is SIA cosine 2ϕ (Figure 5C); therefore,

$$FI = \frac{\text{SIA cosine } 2\phi}{\text{TIA}}$$

The ratio $\frac{\text{SIA}}{\text{TIA}}$ is expressed as the correction index (CI)[6,7]

$$FI = CI \times \text{cosine } 2\phi$$

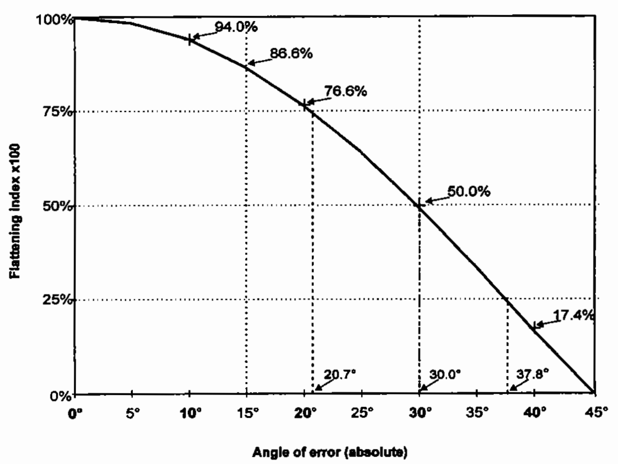


Figure 7A. (Alpins) Effect of misaligned astigmatism treatment on flattening index.

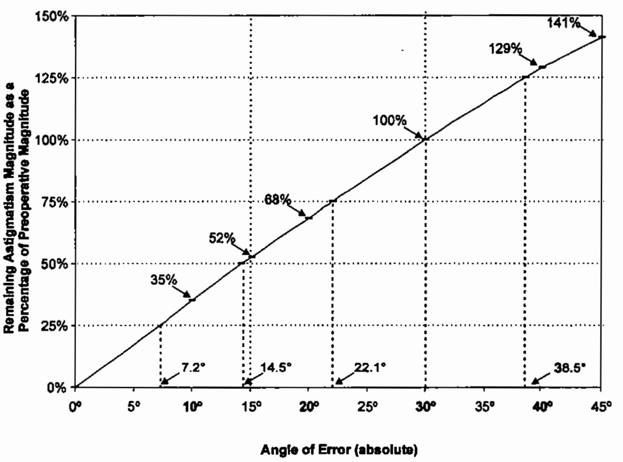


Figure 7B. (Alpins) Effect of misaligned astigmatism treatment on remaining astigmatism magnitude.

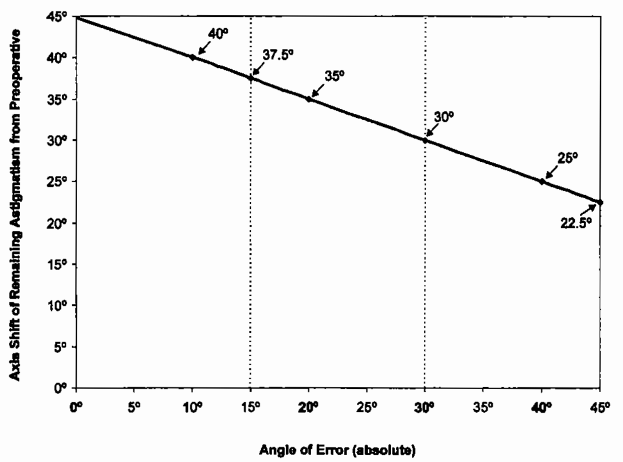


Figure 7C. (Alpins) Effect of misaligned astigmatism treatment on remaining astigmatism axis.

When SIA = TIA, a full correction of astigmatism is achieved, resulting in a correction index of unity. In this case, the determination of the flattening index is simplified to the general relationship:

$$FI = \cos 2\phi$$

where ϕ is the angle of error, or the amount of misalignment between the achieved axis (SIA) and the intended axis (TIA) of treatment.

This FI can be expressed as a percentage if multiplied by 100. In Figure 7A, the FI becomes smaller as a function of increasingly misaligned treatment. In this model, the intended change in astigmatism correction is fully achieved. Thus, any reduction in flattening effect should not necessarily be solely attributed to misaligned treatment because if the SIA is less than the TIA (the correction index is less than 1), it would also contribute to a reduction in the flattening effect and the FI, but by a different mechanism.

The effect of misalignment on the amount of astigmatism remaining (i.e., the surgical residual astigmatism [SRA][7]) is shown in Figure 7B. If the axis misalignment reaches 30 degrees and a full correction of astigmatism is attempted, the preoperative astigmatism magnitude is not reduced. Instead the astigmatism meridian is shifted 30 degrees in the cyclical direction of the treatment.[11,12] This is shown in Figure 2B, in which K_1, TIA, and K_2 are equal. In this case, some flattening occurs (50% of the targeted amount) at the intended meridian (Figure 7A).

Only where the misalignment reaches 45 degrees does the flattening effect at the intended meridian become zero; that is, no measurable change in astigmatism has occurred at this meridian. In this case, the change is purely torque; a shift in the existing astigmatism occurs so that the steepest axis now lies 22.5 degrees in the cyclical direction toward the axis of the steepening treatment. Its magnitude increases by a factor of 1.41 (Figure 7B). This is shown in Figure 2B, where TIA = K_1 and $K_1 = K_2 \cos 45$ degrees so that $K_2 : K_1 = 1.00 : 0.71 = 1.41$.

These variations in flattening effects do not have the near linear relationship with the axis misalignment as suggested by Pender[11] but rather conform to that of a sine wave (Figure 7A). Pender's values for the reduced effect of misaligned astigmatism treatment significantly overstate the impact of this phenomenon when compared with the values determined in this paper.

The axis shift of the remaining astigmatism compared with the preoperative meridian is shown in Figure 7C. When the correction index is unity and the misalignment small, this shift approaches the maximum of 45 degrees when the remaining astigmatism is minimal.[12] When the correction index is close to unity, the larger amounts of axis shift are associated with very small amounts of remaining astigmatism, limiting the impact of what appears to be a large axis change. The values of axis shift and remaining astigmatism have an inverse relationship as seen in Figures 7B and 7C.

Summary

Forces that act on the cornea in an "off-axis" manner have as their component parts some proportion of flattening or steepening effect and clockwise or counterclockwise torque. A clear understanding of the effect of applying pure flattening, steepening, and torque forces on the existing astigmatism of the cornea, and their secondary effect on the refractive astigmatism, enables a better understanding and ultimately enhanced control of corneal shape changes. In this way, any change in the orientation and magnitude of the existing astigmatism can be more effectively achieved.

This paper has sought to address the quantification of the effect of astigmatic changes on astigmatic corneas by identifying the principal astigmatic meridians of the steepest and flattest axes which decrease (or increase) astigmatism, as well as those axes of pure torque effect 45 degrees away. The two formulas required for this analytic method are described.

The flattening effect is introduced as a value, measured in diopters, that provides a measure of the change induced by surgery at its intended orientation. The flattening index is a ratio suggested as a measure of this surgical effect relative to the targeted change in astigmatism quantified by the TIA. This index provides complementary information to the other relative measures, the index of success (Difference Vector/TIA), and the correction index (SIA/TIA) described previously.[6,7]

This method, when used with the astigmatism analysis method previously described for the analysis of vectors,[6] provides a comprehensive understanding of any induced astigmatic change, enabling an integrated

examination of these changes that are applicable to keratometry, topography, or refraction values. When analyzing attempts to reduce or eliminate existing astigmatism with "on-axis" treatment, favorable change for this endeavor is quantified by flattening effect and ineffective change can be evaluated by torque.

Only symmetrical treatments in a cornea with regular topography have been discussed, and the application of these methods in the treatment of nonorthogonal or asymmetrical astigmatism will be addressed in a subsequent paper.[13]

References

1. Cravy TV. Calculation of the change in corneal astigmatism following cataract extraction. Ophthalmic Surg 1979; 10(1):38–49
2. Naeser K. Conversion of keratometer readings to polar values. J Cataract Refract Surg 1990; 16:741–745
3. Holladay JT, Cravy TV, Koch DD. Calculating the surgically induced refractive change following ocular surgery. J Cataract Refract Surg 1992; 18:429–443
4. Naeser K, Behrens JK, Naeser EV. Quantitative assessment of corneal astigmatic surgery: expanding the polar values concept. J Cataract Refract Surg 1994; 20:162–168
5. Grene RB, Lindstrom RL. Astigmatic keratotomy in the refractive patient: the ARC-T study. In: Gills JP, Martin RG, Thornton SP, Sanders DR, eds, Surgical Treatment of Astigmatism. Thorofare, NJ, Slack, 1994; 19
6. Alpins NA. A new method of analyzing vectors for changes in astigmatism. J Cataract Refract Surg 1993; 19:524–533
7. Alpins NA. New method of targeting vectors to treat astigmatism. J Cataract Refract Surg 1997; 23:65–75
8. Jaffe NS. Clayman HM. The pathophysiology of corneal astigmatism after cataract extraction. Trans Am Acad Ophthalmol Otolaryngol 1975; 79:OP615–OP630
9. Naylor EJ. Astigmatic difference in refractive errors. Br J Ophthalmol 1968; 52:422–425
10. Thornton SP, Sanders DR. Graded nonintersecting transverse incisions for correction of idiopathic astigmatism. J Cataract Refract Surg 1987; 13:27–31
11. Pender PM. Photorefractive keratectomy for myopic astigmatism: phase IIA of the Federal Drug Administration study (12 to 18 months follow-up). J Cataract Refract Surg 1994; 20:262–264
12. Stevens JD. Astigmatic excimer laser treatment: theoretical effects of axis misalignment. Eur J Implant Refract Surg 1994; 6:310–318
13. Alpins NA. The treatment of irregular astigmatism. In press, J Cataract Refract Surg

Appendix

Legend for Figures 1 to 6 and Equations

Ax	=	axis
RA	=	reference axis
CW	=	clockwise
CCW	=	counterclockwise
TIA	=	target induced astigmatism vector
SIA	=	surgically induced astigmatism vector
T	=	topography
R	=	refraction
ATR	=	against the rule
WTR	=	with the rule
K_1	=	preoperative astigmatism magnitude
K_2	=	postoperative astigmatism magnitude
θ_1	=	preoperative astigmatism meridian
θ_2	=	postoperative astigmatism meridian
α	=	angle off axis
β	=	axis of SIA
ϕ	=	angle of error
@	=	corneal meridian
••••	=	vector
— —	=	orientation line