

Planning for Coupling Effects in Bitoric Mixed Astigmatism Ablative Treatments

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ABSTRACT

PURPOSE: To demonstrate how to determine the historical coupling adjustments of bitoric mixed astigmatism ablative treatments and how to use these historical coupling adjustments to adjust future bitoric treatments.

METHODS: The individual coupling adjustments of the myopic and hyperopic cylindrical components of a bitoric treatment were derived empirically from a retrospective study where the theoretical combined treatment effect on spherical equivalent was compared to the actual change in refractive spherical equivalent. The coupling adjustments that provided the best fit in both mean and standard deviation were determined to be the historical coupling adjustments. Theoretical treatments that incorporated the historical coupling adjustments were then calculated. The actual distribution of postoperative spherical equivalent errors was compared to the theoretically adjusted distribution.

RESULTS: The study group comprised 242 eyes and included 118 virgin right eyes and 124 virgin left eyes of 155 individuals. For the laser used, the myopic coupling adjustment was -0.02 and the hyperopic coupling adjustment was 0.30, as derived by global nonlinear optimization. This implies that almost no adjustment of the myopic component of the bitoric treatment is necessary, but that the hyperopic component of the bitoric treatment generates a large amount of unintended spherical shift. The theoretically adjusted treatments targeted zero mean spherical equivalent error, as intended, and the distribution of the theoretical spherical equivalent errors had the same spread as the distribution of actual postoperative spherical equivalent errors.

CONCLUSIONS: Bitoric mixed astigmatism ablative treatments may display non-trivial coupling effects. Historical coupling adjustments should be taken into consideration when planning mixed astigmatism treatments to improve surgical outcomes.

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ixed astigmatism ablative treatments are inherently complex because they flatten some parts of the cornea while steepening others. One of the most important considerations is how to correctly target the desired spherical equivalent. This is not trivial, because flattening and steepening ablative treatments of astigmatism produce different coupling effects and thus induce different amounts of spherical shift, usually in opposite spherical directions.

This issue is complicated by the fact that there are different treatment strategies for mixed astigmatism, including bitoric,¹⁻⁷ cross-cylinder followed by spherical equivalent,⁸ and positive cylinder followed by negative sphere.⁹ Bitoric appears to be the most common method because it leads to the minimum amount of ablation,¹⁰⁻¹² so we will restrict our considerations in this article to bitoric treatments of mixed astigmatism.

In 1997, Chayet et al.^{1,2} described a nomogram to account for the spherical shift caused by a bitoric laser ablation treatment of mixed astigmatism using the EC-5000 laser (Nidek, Fremont, CA). This nomogram is predicated on the observation that the myopic (negative) cylinder treatment contributes a hyperopic shift to the keratometric readings but the hyperopic (positive) cylinder treatment contributed no myopic spherical shift. Customized versions of the same nomogram appear to be used by a variety of different lasers made by different manufacturers.^{2,4,5}

Alpins et al.¹³ characterized the coupling characteristics of compound myopic and hyperopic treatments and found that both compound myopic and hyperopic cylinder treatments

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can cause unexpected spherical shifts. Thus, the assumption underlying the Chayet nomogram may not hold in general.

In this article, we show how to design bitoric treatments for the general case, when there are no assumptions about the coupling caused by the myopic and hyperopic cylinder components. We also show how to use historical data to determine appropriate coupling adjustments for the myopic and hyperopic cylinder components of bitoric mixed astigmatism treatments. We then apply these methods to our own clinical data and illustrate the difference in postoperative outcomes that would be expected if we had used historical coupling adjustments to plan our treatments.

PATIENTS AND METHODS

COUPLING EFFECTS OF A BITORIC TREATMENT

For mixed astigmatism, the manifest or wavefront refraction at the corneal plane in positive cylinder format can be written as $-S / C \times A$, where the sphere magnitude S and the cylinder magnitude C are positive, the cylinder axis is A , and S is less than C . The spherical equivalent of the correction is $C / 2 - S$, and thus the desired change in refractive spherical equivalent to reach emmetropia is $S - C / 2$.

In a bitoric treatment, the total treatment comprises a myopic cylindrical component $0 / -C_M \times (A \pm 90)$ and a hyperopic cylindrical component $0 / +C_H \times A$, where C_M and C_H are positive and $C_M + C_H = C$. We used the concept of the coupling adjustment¹³ to quantify the spherical shifts induced by each of these opposing cylindrical components. The coupling adjustment is the amount of sphere per diopter cylinder of astigmatic treatment that needs to be added to a pure cylindrical treatment to counteract the coupling-induced spherical shift. Assume that the coupling adjustments $CAdj_M$ for the myopic component and $CAdj_H$ for the hyperopic component have been determined from historical refractive data. The pure myopic component should then contribute a change in refractive spherical equivalent of:

$$\Delta SEQ_M = C_M/2 + C_M \times CAdj_M = C_M \times (1/2 + CAdj_M)$$

and the pure hyperopic component should contribute a change in refractive spherical equivalent of:

$$\Delta SEQ_H = -C_H/2 - C_H \times CAdj_H = -C_H \times (1/2 + CAdj_H)$$

Thus, the total expected spherical equivalent change is:

$$\Delta SEQ_{\text{expected}} = \Delta SEQ_M + \Delta SEQ_H = C_M \times (1/2 + CAdj_M) - C_H \times (1/2 + CAdj_H)$$

If we substitute $C_H = C - C_M$ into this formula, we get:

$$\Delta SEQ_{\text{expected}} = C_M \times (1/2 + CAdj_M) - (C - C_M) \times (1/2 + CAdj_H)$$

The desired spherical equivalent change is $S - C / 2$. If we choose C_M and C_H to achieve the desired spherical equivalent change, we need to set $\Delta SEQ_{\text{expected}} = S - C / 2$. In this case,

$$S - C / 2 = C_M \times (1/2 + CAdj_M) - (C - C_M) \times (1/2 + CAdj_H)$$

We rearrange this equation to solve for C_M as follows:

$$S - C / 2 = C_M \times (1/2 + CAdj_M + 1/2 + CAdj_H) - C \times (1/2 + CAdj_H)$$

$$S - C / 2 + C \times (1/2 + CAdj_H) = C_M \times (1/2 + CAdj_M + 1/2 + CAdj_H)$$

$$S - C / 2 + C / 2 + CAdj_H \times C = C_M \times (1 + CAdj_M + CAdj_H)$$

$$S + CAdj_H \times C = C_M \times (1 + CAdj_M + CAdj_H)$$

$$C_M = (S + CAdj_H \times C) / (1 + CAdj_M + CAdj_H)$$

This C_M that has just been calculated is the myopic cylindrical component that achieves the desired spherical equivalent change. The hyperopic cylindrical component is then:

$$C_H = C - C_M = C - (S + CAdj_H \times C) / (1 + CAdj_M + CAdj_H)$$

In the trivial case where there is no coupling, $CAdj_M = CAdj_H = 0$, which gives $C_M = S$. Thus, the myopic cylindrical component would be $0 / -S \times (A \pm 90)$ and the hyperopic cylindrical component would be $0 / +(C - S) \times A$, which is the standard split of the original mixed astigmatism treatment into a bitoric treatment.

Example 1: Equal Myopic and Hyperopic Treatments, With Equal Coupling Adjustments. In this example, we use the coupling adjustments $CAdj_M = CAdj_H = -0.30$.

The intended mixed astigmatism treatment is $-1.00 / +2.00 \times 90$, so $S = 1.00$, $C = 2.00$.

$$C_M = (1.00 + [-0.30] \times 2.00) / (1 + [-0.30] + [-0.30]) = 1.00$$

$$C_H = C - C_M = 2.00 - 1.00 = 1.00$$

Thus, the final bitoric treatment is $0.00 / -1.00 \times 180$ and $0.00 / +1.00 \times 90$.

In this simple example, the coupling-induced spherical shifts from the two pure cylindrical treatments cancel out exactly, so the bitoric is exactly the standard split.

Example 2: Unequal Myopic and Hyperopic Treatments, With Equal Coupling Adjustments. In this example, we use the coupling adjustments $CAdj_M = CAdj_H = -0.30$.

The intended mixed astigmatism treatment is $-2.00 / +3.00 \times 90$, so $S = 2.00$, $C = 3.00$.

$$C_M = (2.00 + [-0.30] \times 3.00) / (1 + [-0.30] + [-0.30]) = 2.75$$

$$C_H = C - C_M = 3.00 - 2.75 = 0.25$$

Thus, the final bitoric treatment is $0.00 / -2.75 \times 180$ and $0.00 / +0.25 \times 90$.

Here, we are compensating for an anticipated hyperopic shift by performing a more myopic treatment. The 2.75 diopters cylinder (DC) myopic cylinder treatment contributes $-(2.75 / 2) - (-0.3) \times 2.75 = -0.55$ diopters sphere (DS) to the spherical equivalent change, whereas the 0.25 DC hyperopic cylinder treatment contributes $0.25 / 2 + (-0.3) \times 0.25 = +0.05$ DS to the spherical equivalent change, together giving the desired -0.50 diopters (D) spherical equivalent change.

Example 3: Equal Myopic and Hyperopic Treatments, With Unequal Coupling Adjustments. In this example, we use the coupling adjustments $CAdj_M = -0.05$, $CAdj_H = -0.30$.

The intended mixed astigmatism treatment is $-1.00 / +2.00 \times 90$, so $S = 1.00$, $C = 2.00$.

$$C_M = (1.00 + [-0.30] \times 2.00) / (1 + [-0.05] + [-0.30]) = 0.62$$

$$C_H = C - C_M = 2.00 - 0.62 = 1.38$$

Thus, the final bitoric treatment is $0.00 / -0.62 \times 180$ and $0.00 / +1.38 \times 90$.

In this example, we are expecting a small hyperopic shift from the myopic cylindrical treatment and a large myopic shift from the hyperopic cylindrical treatment, resulting in a net myopic shift. To compensate for that, it is necessary to increase the size of the hyperopic cylindrical treatment and decrease the size of the myopic cylindrical treatment.

DERIVING THE MYOPIC AND HYPEROPIC COUPLING ADJUSTMENTS FOR BITORIC TREATMENTS

Historically, one author (NA) has planned mixed astigmatism surgeries by using an empirically determined nomogram that was specific to a particular laser (AMO

VISX S4 IR; Abbott Medical Optics, Abbott Park, IL). This nomogram is equivalent to setting both the myopic and hyperopic coupling adjustments $CAdj_M$ and $CAdj_H$ to 0.21, and then boosting both the myopic and hyperopic components by 20% to account for an inherent tendency of the inbuilt algorithms to undercorrect cylinder.^{14,15}

We analyzed historical cases treated with this laser between August 2004 and January 2016 with a customized bitoric ablation (the treatment was entered directly into the laser as a plano/hyperopic cylinder: 5- to 9-mm annular optical zone, followed by a plano/myopic cylinder: 6 × 5.5 mm optical zone) to determine which choice of coupling adjustments would have brought the distribution of postoperative spherical equivalents closest to zero. Only patients who had no previous ocular surgery were included, and eyes with keratoconus were excluded. The postoperative data were from the latest postoperative visit that was at least 1 month and no later than 12 months after the surgery. A total of 22% of the surgeries were photorefractive keratectomy procedures and the rest were LASIK procedures. All patients had previously given written consent for the use of their anonymized data in research studies and the procedures followed were in accordance with the tenets of the Declaration of Helsinki of 1975.

Recall that the expected change in spherical equivalent caused by the mixed treatment is

$$\Delta SEQ_{\text{expected}} = C_M \times (1/2 + CAdj_M) - C_H \times (1/2 + CAdj_H)$$

and the actual change in refractive change spherical equivalent is the difference between the preoperative and postoperative spherical equivalents:

$$\Delta SEQ_{\text{actual}} = SEQ_{\text{post}} - SEQ_{\text{pre}}$$

For the treatments that are being analyzed, we happen to know C_M and C_H , because these were explicitly entered into the laser.

The aim here is to have $\Delta SEQ_{\text{expected}}$ to approximate $\Delta SEQ_{\text{actual}}$ as closely as possible. We have two parameters that we can adjust ($CAdj_M$ and $CAdj_H$). We searched the parameter space of $CAdj_M$ and $CAdj_H$ using an exhaustive search (each in the range between -1 and 1, with a granularity of 0.01) to find coupling adjustments where the mean difference between $\Delta SEQ_{\text{expected}}$ and $\Delta SEQ_{\text{actual}}$ was zero and the standard deviation of the difference was low. To be more specific, for each pair of coupling adjustments $CAdj_M$ and $CAdj_H$, the difference $\Delta SEQ_{\text{actual}} - \Delta SEQ_{\text{expected}}$ was calculated for each eye, and then the mean and the standard deviation of this difference were calculated across the whole group of eyes. We took the historical coupling adjustments to

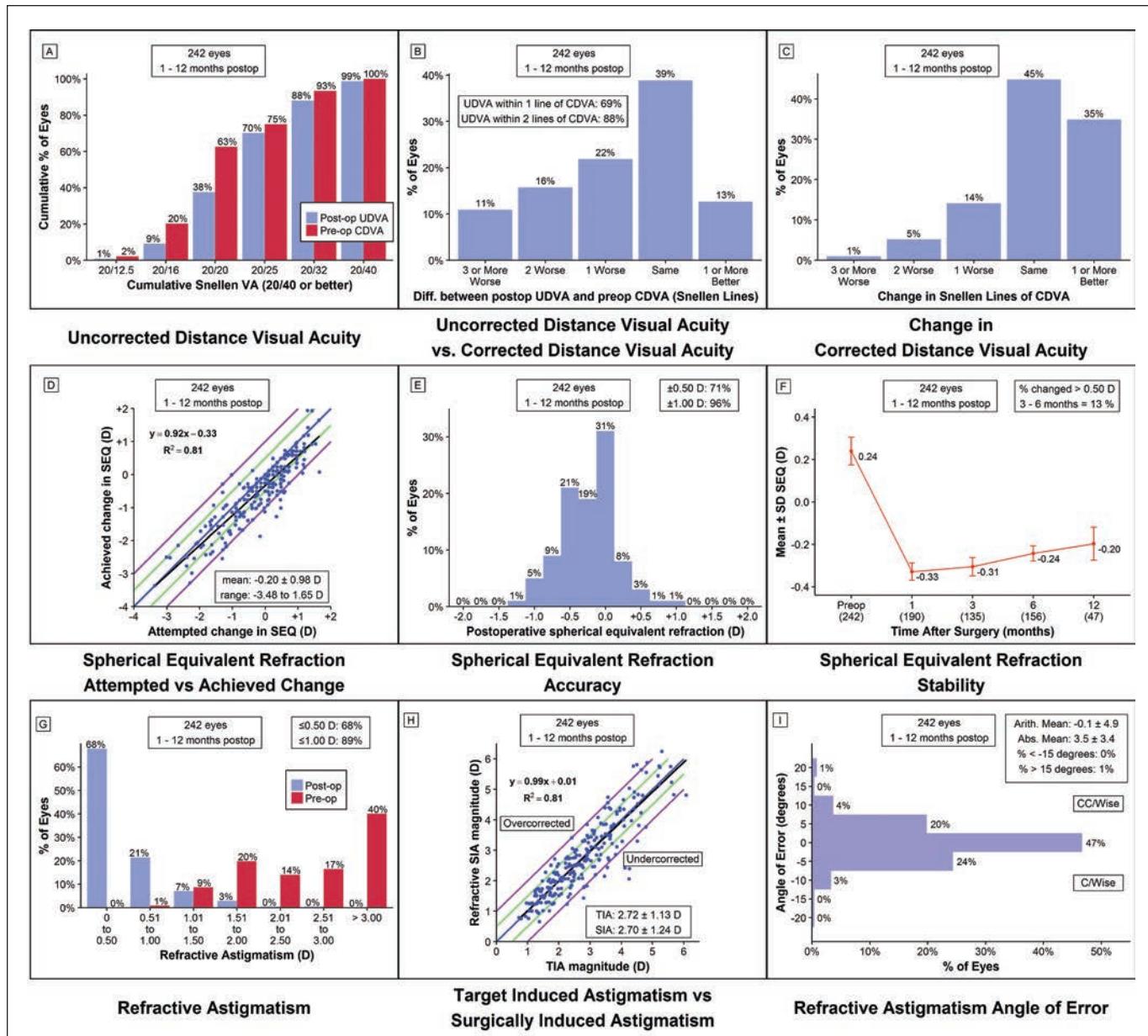


Figure 1. Standard graphs for reporting refractive surgery outcomes. D = diopters; UDVA = uncorrected distance visual acuity; CDVA = corrected distance visual acuity; SEQ = spherical equivalent refraction; SIA = surgically induced astigmatism vector; TIA = target induced astigmatism vector

be the pair of coupling adjustments that had the mean difference with magnitude less than 0.01 and the minimum possible standard deviation of the difference.

We also determined the new distribution of spherical equivalent error if we had used these coupling adjustments to plan the bitoric treatments.

All analysis was performed in R software (The R Foundation, Vienna, Austria).

RESULTS

This study assessed data in 242 eyes: 118 virgin right eyes and 124 virgin left eyes of 155 individu-

als (87 men and 68 women; age range: 20 to 62 years, mean \pm standard deviation [SD]: 39 ± 10 years). All bitoric treatments targeted a postoperative refractive spherical equivalent between emmetropia and 0.50 D myopia. The spherical equivalent of the intended bitoric treatment at the corneal plane ranged between -1.78 and +2.89 D (mean \pm SD: 0.07 ± 0.95 D), and the total intended treatment cylinder at the corneal plane ($C = C_M + C_H$) ranged between 0.72 and 6.07 D (mean \pm SD: 2.72 ± 1.13 D).

The standard graphs for reporting refractive outcomes are shown in Figures 1-2. Figure 1D shows that

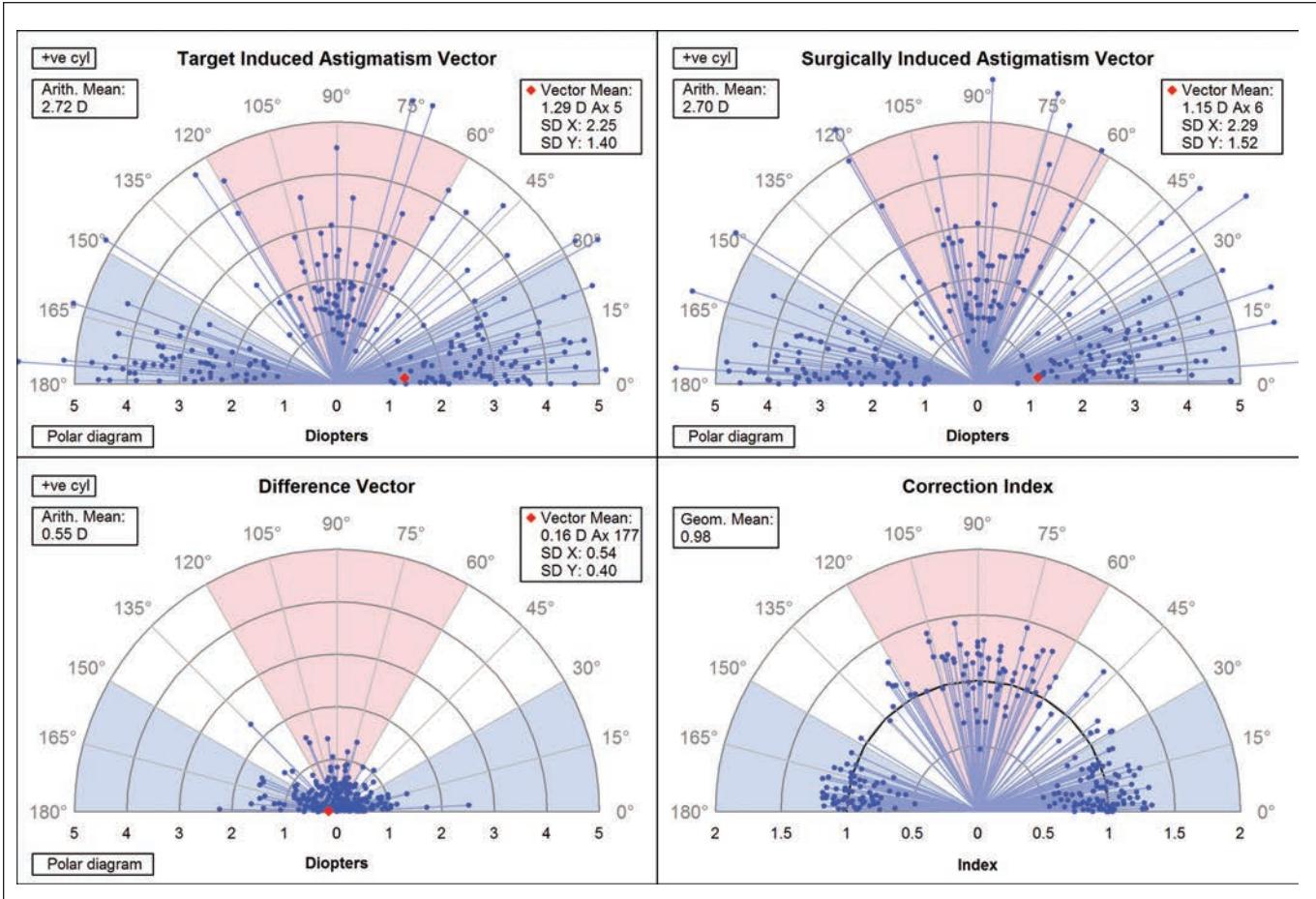


Figure 2. Standard graphs for reporting outcomes for astigmatism correction, based on the Alpins method. D = diopters; SD = standard deviation

negative changes in SEQ are generally correct, whereas positive changes in SEQ appear to be undercorrected. This explains the slight bias toward a myopic postoperative refraction shown in **Figures 1E-1F**, and is consistent with the less-than-optimal postoperative visual acuities in **Figures 1A-1B**. **Figure 1H**, which compares the refractive surgically induced astigmatism¹⁶ to the target induced astigmatism,¹⁶ shows that the 20% boost in both the myopic and hyperopic cylindrical components compensated adequately for the undercorrection of cylinder inherent to this particular laser; the gradient of the best fit line passing through the origin, also known as the correction index,¹⁸ is 0.99 (not significantly different from one, $P = .42$). These good astigmatic outcomes are also reflected in **Figure 2**. For the rest of the analysis in this article, we ignore the explicit need for a 20% boost and assume that the required amount of cylindrical treatment is actually being carried out.

Figure A (available in the online version of this article) shows plots of the mean and the standard deviation of the difference between $\Delta\text{SEQ}_{\text{expected}}$ and $\Delta\text{SEQ}_{\text{actual}}$ across the parameter space of coupling ad-

justments. The coupling adjustments that best describe the historical treatments are those that produce a mean difference of zero, with the lowest possible standard deviation of the difference. To find the coupling adjustments that satisfy these criteria, we begin by restricting our search to those points where the mean difference between the expected and the actual change in spherical equivalent is zero (see **Figure A**, left graph). This turns out to be a straight line in the parameter space. Now, we look for the minimum standard deviation of the difference between $\Delta\text{SEQ}_{\text{expected}}$ and $\Delta\text{SEQ}_{\text{actual}}$ for all points on this line. This minimum occurs for coupling adjustments $C\text{Adj}_M = -0.02$, $C\text{Adj}_H = 0.31$, and the value of the standard deviation of the difference there is only 0.02 D more than the global minimum (see **Figure A**, right graph, and **Figure B**, available in the online version of this article).

Using these coupling adjustments, we recalculated the correct bitoric split of the original mixed astigmatism treatments, targeting the same spherical equivalent target as the original treatment. We then adjusted the postoperative spherical equivalent error (the difference

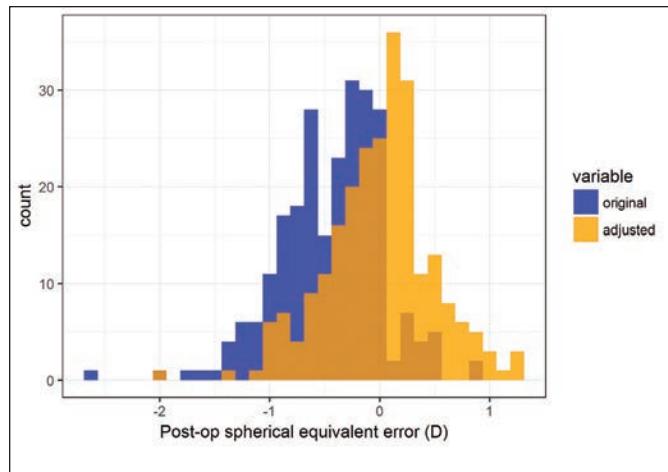


Figure 3. Original histogram of postoperative spherical equivalent error, overlaid with the adjusted histogram from selecting a new bitoric split using the new coupling adjustments. The adjusted histogram is centered on zero spherical equivalent error and has approximately the same amount of spread as the original histogram. D = diopters

between the spherical equivalent target and the postoperative spherical equivalent [ie, $\text{SEQ}_{\text{post}} - \text{SEQ}_{\text{target}}$] by the difference in expected spherical equivalent change between the old treatment [$\Delta\text{SEQ}_{\text{expected}}$ using $\text{CAdj}_M = 0.21$, $\text{CAdj}_H = 0.21$] and the new treatment [$\Delta\text{SEQ}_{\text{expected}}$ using $\text{CAdj}_M = -0.02$, $\text{CAdj}_H = 0.31$]. **Figure 3** shows a histogram of the spherical equivalent error, overlaid with the adjusted spherical equivalent error. The adjusted histogram is now centered on zero error (old mean: -0.44 D, new mean: 0.00 D), and has approximately the same amount of spread as the old histogram (old SD: 0.48 D, new SD: 0.49 D).

DISCUSSION

We have shown how to derive historical coupling adjustments for bitoric treatments where the division into myopic and hyperopic cylindrical components is known. We have also shown how to adjust bitoric treatments when historical coupling adjustments are known. Adjustment of bitoric treatments for coupling is necessary to avoid systematic error in the postoperative spherical equivalent. Normally, this adjustment would be performed by laser manufacturers in the device nomogram, but surgeons performing many treatments of mixed astigmatism should also be able to adjust their own bitoric treatments by adjusting the treatment nomogram.

The exact bitoric split of mixed astigmatism treatments into myopic and hyperopic cylindrical components should first be determined to be able to perform a historical analysis, even when such treatments are entered by the surgeon in an overtly spherocylindrical form. Such a historical analysis could then be considered in the planning of future mixed astigmatism treatments.

When the exact mixed astigmatism treatment paradigm is unknown, a reasonable starting point might be to postulate that the ablation strategy is either bitoric or a sequential combination of myopic sphere and hyperopic cylinder. The postoperative results should then be analyzed according to both treatment strategies: a bitoric analysis would follow the procedure described in this article, whereas an analysis of a combined myopic sphere and hyperopic cylinder has been described in a previous study.¹³ The closest fit should indicate which treatment strategy has been used. In the case where a bitoric strategy seems to have been used, both myopic and hyperopic coupling adjustments can be calculated. For the case of a sequential combination of myopic sphere and hyperopic cylinder, the spherical treatment efficacy and the hyperopic coupling adjustments need to be calculated.

If it has been determined that the laser software generates the ablation using a bitoric approach but the desired treatment is entered as a spherocylindrical refraction, the treatment to be entered into the laser software is the simple sum of the myopic and hyperopic cylindrical components derived after the application of the coupling adjustments, converted to spherocylindrical form. The easiest way to do this is to transpose one of the cylinder components into its spherocylindrical equivalent so that the two cylinder axes now match, then add the cylinders.

One interesting detail that arises from the analysis of our data is that Chayet et al.'s assumption does not hold (ie, the hyperopic component may actually contribute a coupling effect). There are several reasons for this outcome. The most important reason for this is that our adjustment is applied over and above the nomogram used in the laser; effectively, it is an adjustment of an adjustment. Also, we used refractive change instead of keratometric change. It has been seen previously that findings about coupling differ greatly between those derived from refraction and those derived from keratometry.¹³ This makes sense because a hyperopic ablation is likely to occur outside of the zone where keratometry has traditionally been measured, thus having a minimal impact on keratometry but a large effect on refraction. Finally, it is possible that blend zones between the two toric components may cause some spherical shift.

In this study, we have assumed that the surgeon already has a nomogram to account for any consistent astigmatic undercorrection or overcorrection. In our case, this nomogram was a boost of 20% for both the myopic and hyperopic cylinder. We recommend applying this nomogram after the coupling adjustments have been applied because it is conceptually simpler:

the effect of the nomogram causes the laser to treat the intended amounts of myopic and hyperopic cylinder.

We acknowledge that there are limitations with this study. It would have been desirable to have used results from a more constrained time period with a higher follow-up rate instead of using data from treatments that occurred over the span of more than a decade with the same laser. Also, the results need to be validated by prospective treatments using the new coupling adjustments. **Figure 1C** shows a loss of two or more lines of corrected distance visual acuity in 6% of cases, which can be attributed to the inclusion of photorefractive keratectomy procedures with 1-month postoperative data. In addition, the results from this study cannot be directly extrapolated to other lasers, or even to the same laser using a different treatment paradigm or profile. However, we still believe that the results illustrate how to compensate for inherent systematic errors in mixed astigmatism treatments caused by coupling.

The ultimate goal of a mixed astigmatism treatment is to have no unintended spherical shift. Even when the preoperative spherical equivalent is zero and the hyperopic and myopic astigmatic treatments are numerically equal in size, we have seen that the effects on the spherical equivalent may not cancel out. When there is the expectation of unintended spherical shift due to coupling, the treatment must be adjusted accordingly. However, it is preferable to avoid the overt use of an additional spherical treatment because this will invariably increase the amount of ablated tissue. We have shown how to choose an appropriate mixed astigmatism treatment to target the correct spherical equivalent change.

AUTHOR CONTRIBUTIONS

Study concept and design (NA, JKYO); data collection (JKYO, GS); analysis and interpretation of data (NA, JKYO, GS); writing the manuscript (JKYO, GS); critical revision of the manuscript (NA, JKYO); statistical expertise (JKYO); administrative, technical, or material support (NA, GS); supervision (NA, GS)

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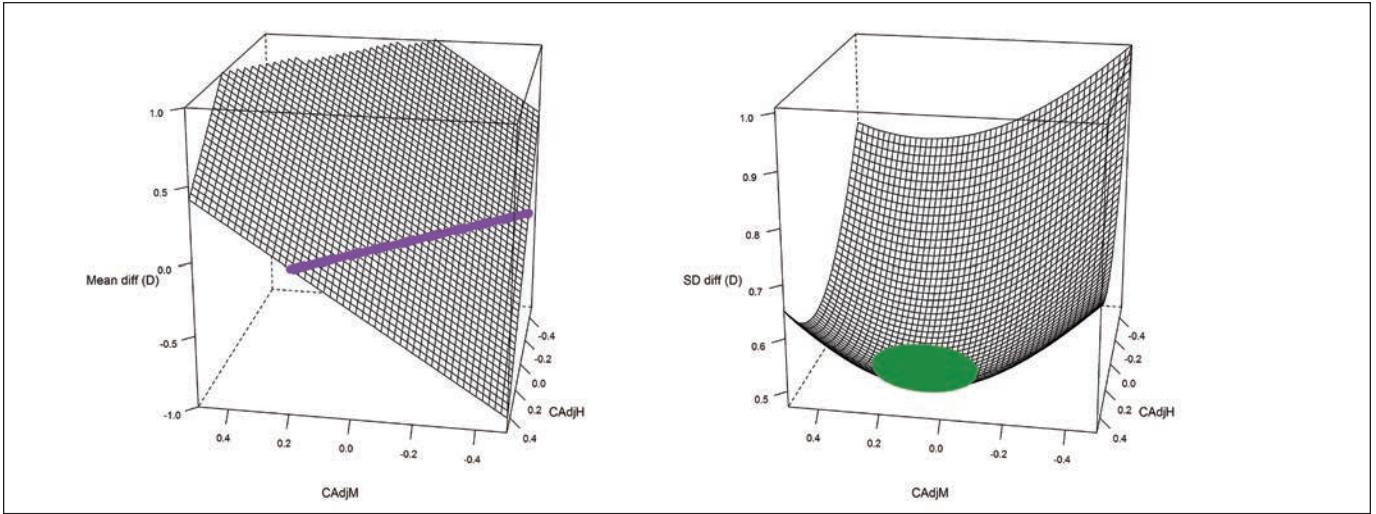


Figure A. The difference between the expected and the actual change in spherical equivalent depends on the values of the coupling adjustments $CAdj_M$ and $CAdj_H$. The mean of this difference over all eyes is shown in the left graph and the standard deviation of this difference is shown in the right graph. The dark line on the left graph corresponds to the coupling adjustments that result in a mean difference of zero. The dark area on the right graph, centered at $CAdj_M = -0.02$, $CAdj_H = 0.31$, corresponds to the coupling adjustments where the difference has a standard deviation no more than 0.02 diopters (D) above the minimum. SD = standard deviation; $CAdj_M$ = myopic cylinder coupling adjustment; $CAdj_H$ = hyperopic cylinder coupling adjustment

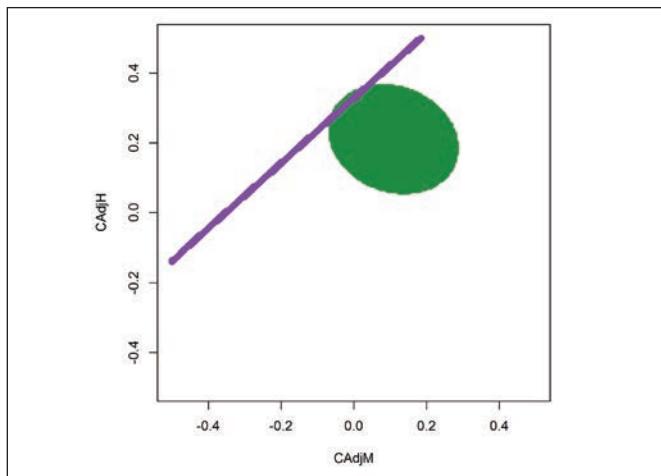


Figure B. The parameter space of coupling adjustments, showing the coupling adjustments that produce zero mean difference between the expected and actual spherical equivalents (line) and the coupling adjustments that produce a low standard deviation of the difference (ellipse). The combination of coupling adjustments $CAdj_M = -0.02$ and $CAdj_H = 0.31$ has a difference with zero mean and a low standard deviation. $CAdj_M$ = myopic cylinder coupling adjustment; $CAdj_H$ = hyperopic cylinder coupling adjustment